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DESCRIPTIVE GEOMETRY FOR ENGINEERS

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CLARENCE E. TOWNSEND

Introductory Mechanical Drawing

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Drawing Room Problems

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BY

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PREFACE

The scope of this text is restricted to a treatment of those elements of descriptive geometry which form the basis of drawing as applied in engineering.

For purposes of presentation, the subject is classified in accordance with the functions assigned to graphic representation in mechanical drawing. *Graphic description* naturally received first attention and is treated at greater length than *graphic computation.*

The basic principles of the system of graphic description are developed from a fundamental consideration of a *point* and a *primary pair of reference planes.* These principles are then applied to the description of lines, planes and solids and to the description of their in-space relations.

The brief treatment of graphic computation serves to identify this capacity of graphic representation with design and paves the way toward practical drawing-board work in simple design problems. It also provides an immediate and natural application for the principles of graphic description.

A set of recitation questions is included as an essential part of the text. These questions are designed to guide the student and also to supplement the text proper by providing indirect instruction.

Exercises and problems covering both fundamental principles and practical drawing-board work may be secured from the publisher.

The author is indebted to Mr. G. R. Hanselman and Mr. L. A. Fenner, co-members of the department of drawing at Cornell, for helpful suggestions, and especially to Professor C. E. Townsend, head of the department of drawing, who has read the manuscript.

S. F. CLEARY.

ITHACA, N. Y.
January, 1930.

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DESCRIPTIVE GEOMETRY

CHAPTER I

INTRODUCTION

In presenting this subject it is assumed that the student is familiar with the general natures of the various types of graphic representations and with the purposes for which they are employed in design and in working drawing. It is further assumed that some facility has been acquired in the use of drafting-room equipment and in the execution of lettering and delineation.

Descriptive geometry, the subject of this study, is the science upon which all types of graphic representations are based. Difficulties already experienced in reading and in making working drawings may, in the estimation of many students, amply justify an investigation of this science. However, a knowledge of descriptive geometry is of vastly greater importance in design drawing than in working drawing, because the designer must employ graphic representation as a means of *solving* problems involving size and shape, whereas in working drawing graphic representation is employed merely as a means of *describing* objects whose size and shape have already been determined.

The benefits which may be derived from a study of descriptive geometry are by no means confined to those which bear only upon mechanical drawing. Study of this subject affords an early opportunity for training in analytical reasoning; in fact, development of the ability to reason analytically is an essential factor in the mastery of the subject. An opportunity as well as an incentive to develop other engineering traits is also offered, notable among them the ability to think and visualize in three dimensions — an ability of utmost importance to the designer.

CHAPTER II

BASIC PRINCIPLES OF GRAPHIC DESCRIPTION

1. The Descriptive Unit. — In explaining the science of descriptive geometry it is first necessary to establish the identity of the unit by means of which objects are graphically described. Students of elementary mechanical drawing are already familiar with the fact that in working drawing objects are graphically described by representations of their *characteristic lines* and that these lines include those forming the silhouette of the object and those created by the intersection of the object's surfaces. The *line*, however, is not the descriptive unit, since it may be divided into points which, if represented in total, result in a representation of the line. Obviously, since a *point* cannot be further divided it must be considered the unit of description.

Recognition of the point as the descriptive unit correctly infers that the entire scheme of graphic description is based upon a method of graphically describing this unit.

2. The Rudimentary Scheme of Point Description. — A point is non-dimensionable; that is, it has neither length, breadth nor thickness; its *location* is therefore the only factor involved in its description.

Space being of three infinite dimensions, the location of a point cannot be specified until a datum has been established for each of these dimensions. Three planes called *reference planes* or *datum planes* are assumed for the purpose of supplying these data. Since the three dimensions of space are measured mutually at right angles these three reference planes must also be mutually perpendicular.

The following example illustrates in a general manner the method of employing three reference surfaces for specifying the position of a point in space. For the given data of this example assume a lamp suspended in a rectangular

room, see Fig. 1. Let it be required to obtain a record of the lamp's location in the room; such that if the lamp were removed it could, by virtue of the recorded information, be replaced in its original position. To obtain such a record select three adjacent surfaces of the rectangular room, say the floor and two adjacent walls, and consider them as reference planes. If the distances from the lamp to each of these three surfaces are measured, see Fig. 2, and their values recorded, the record thus obtained would incorporate

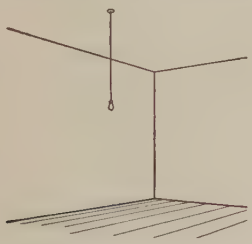


Fig. 1. Data for Example of Point Location.

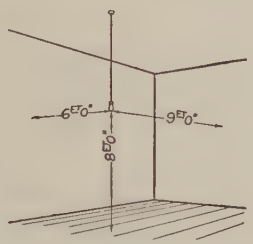


Fig. 2. Locating a Point in Space with Reference to three Datum Planes.

the desired positioning information. It should be noted that any three adjacent surfaces of the rectangular room may be selected as reference planes and that the distance from the lamp to a reference plane must be measured along a perpendicular from the lamp to the plane. It should further be noted that a minimum of three reference planes and three corresponding dimensions are required to fix the location of the lamp.

The foregoing example merely illustrates the general principles of the method established for fixing the location of a point in space. It is now necessary to show how this method is applied when making *graphic* records of the positions of points. Again considering the lamp problem, assume that points on the floor and walls are marked where perpendiculars from the lamp strike them, see Fig. 3. If the lamp is removed after thus marking points in the reference surfaces, the information recorded by the marks would

obviously define the lamp's original position, see Fig. 4. Evidently the three dimensions necessary to locate the lamp are *graphically* recorded by the marks. Investigation shows

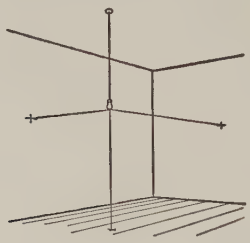


Fig. 3. A Method of Graphically Recording the Position of a Point.

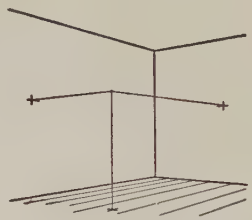


Fig. 4. A Graphic Record of the Position of a Point.

this to be true; the mark on the floor, by virtue of its distances from the intersections of the floor with the walls, specifies the distance from the lamp to each of the walls; likewise the mark on either wall specifies the distance from the lamp to the floor and also to the other wall, see Fig. 5. Two observations of prime importance to the student result from this investigation. The first is, *that the graphic record borne by any one reference plane specifies the distance of the point under consideration from all other reference planes which are perpendicular to it.* The second is that, *the combined graphic records borne by any two adjacent and perpendicular reference planes specify the three dimensions necessary to fix the position of the point in space.*

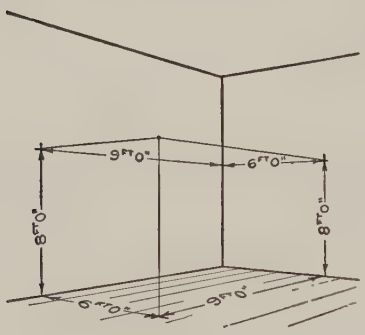


Fig. 5. How a Graphic Record Expresses the Positioning Dimensions of a Point.

3. Adaption of Point Description to the Drawing Board. — To complete the explanation of the general scheme employed in practical graphic description it remains but to show how

the information necessary to the specification of the location of a point in space may be graphically expressed on a single plane surface corresponding to a sheet of drawing paper.

Returning again to the lamp example, assume that floor and walls are replaced by reference planes and the lamp is replaced by a point A, see Fig. 6. Next project the point *orthographically*[1] upon each of the three reference planes.

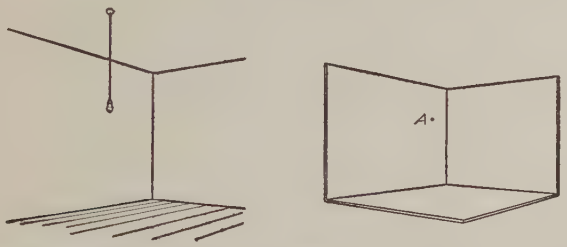


FIG. 6. Comparison of the Actual Reference Surfaces Used in the Lamp Example to the Assumed Reference Planes Used in Practical Graphic Description.

The representations of the point resulting from these projections, see Fig. 7, correspond to the marks made on the floor and walls in the previous consideration of this problem.

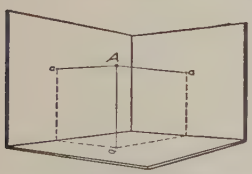


FIG. 7. A Point in Space and its Representations on Reference Planes.

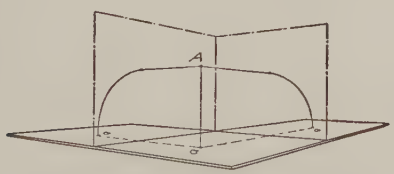


FIG. 8. Formation of a Single Surface Graphic Description of the Position of a Point.

Obviously these representations of the point describe its position relative to the reference planes in the same manner

[1] The *orthographic projection* of a point upon a plane is performed by a projection ray perpendicular to the plane and passing through the point. A *representation* of the point is created by the ray striking the plane. The representation thus created is often called a *projection of the point*, and the plane a *plane of projection*.

as the marks described the lamp's position relative to the floor and walls.

If the reference planes are now folded about their lines of intersection as axes, see Fig. 8, so as to form a single plane surface, corresponding to that of a sheet of drawing paper, this surface will still bear the information necessary to define the position of the point in space.

Actual drawings made in accordance with this scheme, see Fig. 9, must show those intersecting lines of the reference planes which were used as axes during the operation of folding, since it is only by their relation to these lines that the representations of the point are enabled to specify the point's position. Each representation of a point shown on the drawing should be thought of as a specific graphic statement, telling the distance from the actual point in space to all planes which are perpendicular to the one bearing the representation.

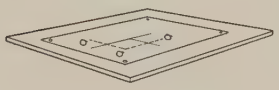


FIG. 9. The Position of a Point Described on a Practical Drawing.

It should again be observed that the two representations borne by any *pair of reference planes* are sufficient to specify the three dimensions which fix the position of a point in space, but that all three of these dimensions must be known before such a pair of representations may be located when making a drawing. A *pair of reference planes* consists of any two perpendicular reference planes which are folded about their line of intersection during the process of forming the single plane surface corresponding to the drawing sheet. Two reference planes which are not perpendicular cannot be used as a pair of reference planes.

It should further be noted at this time that the two representations of a point which are borne by a pair of reference planes must be shown by the drawing on a line perpendicular to the intersection of the planes; the reason for this arrangement may be easily established by investigating the method of projecting upon the planes and the manner of folding them into the drawing surface.

CHAPTER III

THE SYSTEM OF GRAPHIC DESCRIPTION

4. Introduction. — The science of descriptive geometry has been created by the formulation of a suitable system for applying the basic principles of graphic description introduced by the preceding chapter. The present chapter is devoted to a detailed study of the elements of this system.

5. The Primary Pair of Reference Planes. — Since reference planes must be treated in pairs during the operation of folding them and also since but a single pair of these planes is required for the complete description of a point, the laws governing the established method of applying the basic principles of graphic description have been formulated from a fundamental consideration of the relation of a point to a single pair of reference planes. The pair of planes employed for this fundamental consideration is called the *primary pair of reference planes*. Because it is conventional to conceive two of the dimensions of space as being respectively vertical and horizontal, the primary planes are assumed one in a vertical and the other in a horizontal position. The names established for these two primary planes are based upon their assumed positions, one being known as the *primary vertical reference plane*[1] and the other as the *primary horizontal reference plane*, see Fig. 10.

The primary reference planes are considered to be of

[1] A variety of terms are in general use for designating the primary pair of reference planes, the most used of which are: the principal reference planes; the principal planes of projection; the principal projection planes; the principal planes; the V- and H-planes, etc.

The primary horizontal reference plane is known by several names such as: the horizontal reference plane; the horizontal plane of projection; the H-plane; etc.

The primary vertical reference plane like the horizontal is often referred to as: the vertical reference plane; the vertical plane of projection; the V-plane; etc.

limitless extent; they therefore divide space into four parts, one part being subtended by each of the four dihedral angles formed by the intersecting planes. Each of these dihedral angles is formed by a combination of different portions of the primary planes. Since a point in space may be located in any one of the four dihedral angles and since its position must be described with reference to different portions of the primary planes for each angle, it is impera-

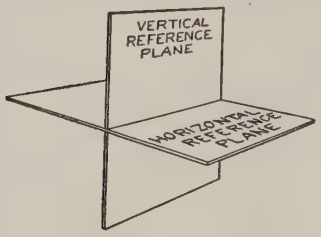


FIG. 10. Arrangement of the Primary Pair of Reference Planes.

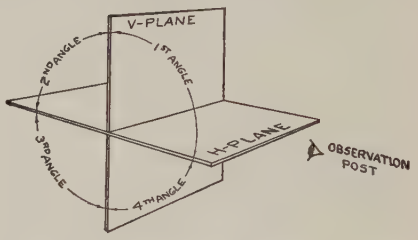


FIG. 11. Naming the Primary Dihedral Angles.

tive that the established system of description be capable of describing points in all four angles without confusion. All the fundamental elements of the system are involved in meeting this requirement. These elements, listed below will be given detailed treatment on the following pages.

1. The method of naming the dihedral angles formed by the primary reference planes.
2. The method of projecting representations upon the primary reference planes.
3. The method of notating the representations.
4. The method of folding the primary planes to form the surface of the drawing.

6. The Method of Naming the Dihedral Angles. — The four dihedral angles are named with reference to an established observation post, assumed to be in front of the vertical reference plane. The angles are assigned identifying numbers relative to this assumed post of observation as explained by the following tabulation and illustrated by Fig. 11.

1st Angle above H and in front of V
2nd Angle above H and in front of V
3rd Angle below H and in back of V
4th Angle below H and in front of V

7. The Method of Projecting Upon the Primary Planes. — In accordance with the basic principles already discussed all representations shown on reference planes must be obtained by means of orthographic projection. However, the position of the observer when performing the act of projecting must now be considered.

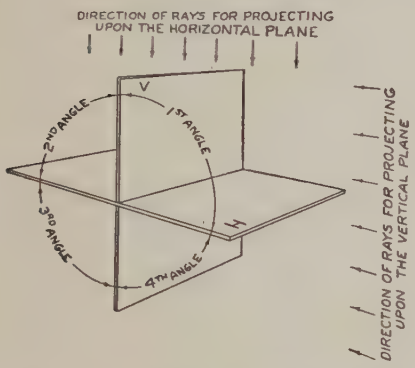


FIG. 12. Direction of Projecting upon the Primary Pair of Reference Planes.

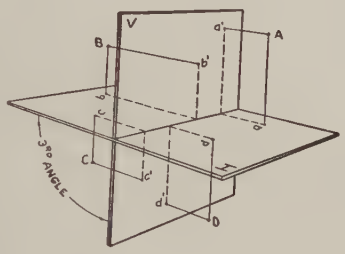


FIG. 13. Method of Notating the Representations of Points Projected upon the Primary Pair of Reference Planes.

When projecting upon the horizontal reference plane the established position of the observer is above the plane regardless of the angle in which the point being described is located. When projecting upon the vertical reference plane the established position of the observer is always in front of the plane, see Fig. 12.

8. Notation of Primary Representations. — An actual point in space is designated by a capital or upper case letter. The representations of such a point are assigned the same letter as the point itself but of lower case type, see Fig. 13.

To distinguish the representation borne by the V-plane[1] from that shown on the H-plane, the vertical representation is primed, see Fig. 13.

9. Folding the Primary Planes. — The established method of folding the primary planes to obtain the single plane surface corresponding to the surface of a drawing, is to open the first dihedral angle by rotating the reference planes about their intersection, see Fig. 14. The result of thus folding the planes is that the upper portion of the V-plane

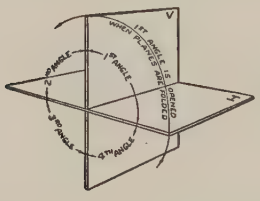


FIG. 14. The Established Direction of Folding the Primary Pair of Reference Planes when Forming the Drawing Surface.

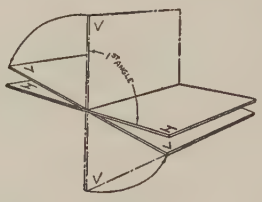


FIG. 15. Folding the Primary Pair of Reference Planes.

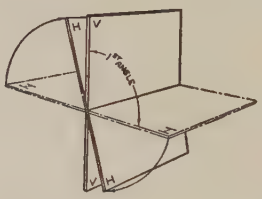


FIG. 16. Folding the Primary Pair of Reference Planes. This method results in the same Arrangement as that shown in Fig. 15.

coincides with the back portion of the H-plane and the lower portion of the V-plane coincides with the front portion of the H-plane. To facilitate visualization of the act of folding, either plane may be assumed as stationary and the other revolved about their intersection, see Figs. 15 and 16.

10. Execution of Point Drawings. — (*a*) The first step in executing an actual drawing is to assume a narrow horizontal line representing the intersection of the primary pair of reference planes. This line is called the *primary ground line* and is labeled as shown in Fig. 17.

(*b*) The next step is to mentally determine the locations,

[1] Representations projected upon the primary vertical reference plane are referred to by a variety of terms such as: vertical representations; vertical projections; V-representations; V-projections; etc. Similar terms are used to designate representations projected upon the primary horizontal reference plane.

relative to the gound line, of the two primary representations of the point. These locations are dependent upon two factors: first, the dihedral angle in which the point is situated; and second, the distance from the point to each of the reference planes.

For example, let it be desired to describe a point which is 1 inch above H and $\frac{1}{2}$ inch in front of V. The fact that this point is above H and in front of V fixes its position as in the first angle. Because the point is in the first angle it is known that its representation on the V-plane will be

FIG. 17. Labeling the Primary Ground Line.

projected upon that portion of the V-plane which is above H. After folding the planes to form the drawing surface, this V-representation will be shown on the drawing above the ground line. By following a similar system of reasoning it will be seen that the H-representation will, in this case, be shown on the drawing below the ground line.

Since the representation of a point on any reference plane is a graphic statement to the effect of its distance from all other reference planes perpendicular to the one on which it is shown, the V-representation of this point will be shown on the drawing 1 inch above the ground line because the actual point is located 1 inch above the H-plane. The H-representation of this point will therefore be shown $\frac{1}{2}$ inch below the ground line since the actual point is $\frac{1}{2}$ inch in front of the V-plane.

(*c*) The third step consists of executing the actual delineation of the drawing. First a line perpendicular to the ground line must be established on which to locate the representations. When attempting to establish this line it is found that the information available is not adequate, since but two of the three dimensions necessary to locate

a point in space have been given. This difficulty vanishes, however, when it is realized that the third reference plane used in defining position may be added to suit the position of the point as described by the original pair of planes. The primary representations may therefore be shown on the drawing at any desired distance along the ground line. To assist the student with the arrangement of drawings this distance along the G. L. from the left border of the paper will hereafter be specified as a part of the given data for problems.

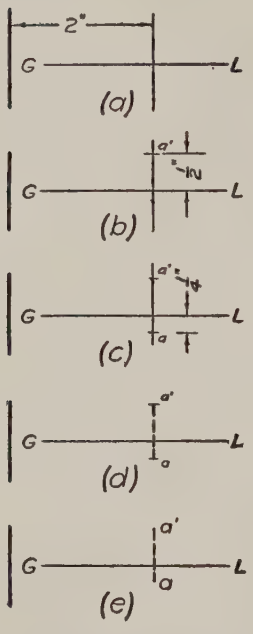


Fig. 18. The Steps of Executing a Drawing which Describes the Position of a Point.

Assuming for the problem under consideration that this distance is 2 inches, a light *measuring line* may be erected on which to locate the representations of the point, see Fig. 18*a*. The representations should be lightly notated when they are first located on this measuring line, see Figs. 18*b* and 18*c*. After locating the representations they should be connected with a *ruled projector*, see Fig. 18*d*, and the drawing finished by replacing the temporary notation with neatly executed letters and by erasing any superfluous construction work, see Fig. 18*e*.

The ruled projector, unlike a measuring line, is a required line of the finished drawing and should therefore be drawn equal in color to other required lines such as the ground line. It is formed by dashes and spaces; the dashes vary from $\frac{1}{16}$ to $\frac{1}{8}$ inch and the spaces from $\frac{1}{32}$ to $\frac{1}{16}$ inch in length, see Fig. 19. The ruled projector serves with the notation to identify pairs of representations of a point; notation and ruled projectors should therefore possess the same degree of visibility and, because they are referred to

during the process of reading a drawing before the ground line, this visibility should be somewhat greater than that

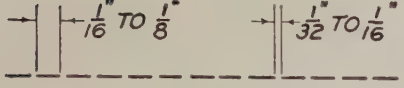


Fig. 19. The Ruled Projector.

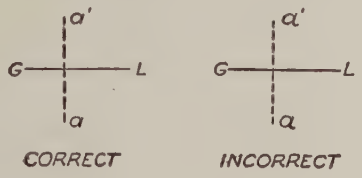
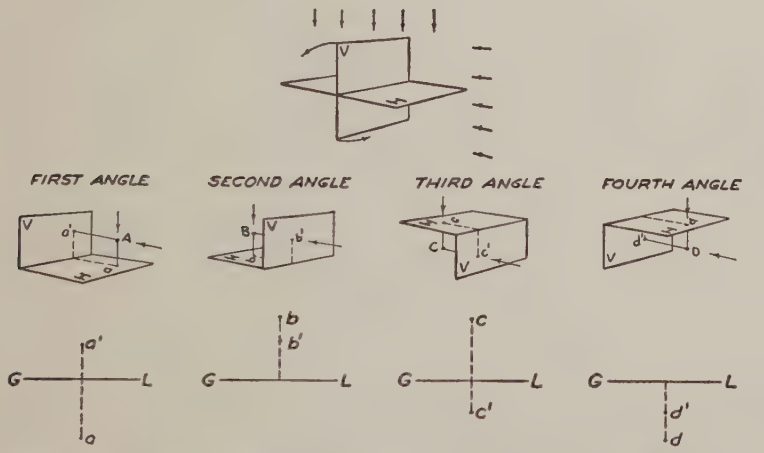


Fig. 20. Delineation.

assigned the G. L. They are, therefore, drawn somewhat wider than the G. L., see Fig. 20, and notation is given a corresponding visibility.



ANGLE	ORDER OF PROJECTING		METHOD OF FOLDING	FINAL POSITION OF REPRESENTATIONS	
	ON H	ON V		V	H
FIRST	EYE, OBJECT, PLANE	EYE, OBJECT, PLANE	OPENED	ABOVE G.L.	BELOW G.L.
SECOND	EYE, OBJECT, PLANE	EYE, PLANE OBJECT	CLOSED	ABOVE G.L.	ABOVE G.L.
THIRD	EYE, PLANE, OBJECT	EYE, PLANE, OBJECT	OPENED	BELOW G.L.	ABOVE G.L.
FOURTH	EYE, PLANE, OBJECT	EYE, OBJECT, PLANE	CLOSED	BELOW G.L.	BELOW G.L.

Fig. 21. Characteristics of the Four Angles of Graphic Description.

11. Types of Graphic Descriptions. — Due to the established methods of projecting upon the primary reference planes and of folding these planes into the drawing sheet,

the particular dihedral angle in which a point is located is clearly indicated by a drawing; in fact a distinct type of graphic description exists for each of the four dihedral angles, see Fig. 21. These four types of graphic descriptions are called *first angle descriptions, second angle description,* etc., dependent upon the dihedral angle in which the points or other data to be described are located. The characteristic features of each of the different types may be readily determined by tracing the application of the established methods of projecting upon and of folding the primary pair of planes. Fig. 21 illustrates the procedure of making such an investigation and lists the more important of the findings.

12. Additional Reference Planes. — The rules established for the four types of graphic description through consideration of the primary pair of reference planes were devised for the purpose of governing the use of every pair of planes employed for reference purposes. When an additional reference plane is employed it is simply necessary to recognize that a new pair of reference planes is created by it and an existing plane and to then apply the established rules of graphic description to this new pair. The arrangement of the additional plane relative to existing planes and the application of the established methods of projecting upon and folding the reference planes for the particular dihedral angle in which the new plane is used are the chief factors to be considered.

The first consideration is that of arrangement. The only restriction in this respect is that the new plane must form a *pair of reference planes* with an existing plane as a mate. It should be borne in mind that a *pair of reference planes* must be at right angles to each other and that they must be folded about their intersection when forming the surface of the drawing paper.

As many additional planes as may be desired can be used with either of the primary planes as a mate, see Fig. 22. An established additional plane can in turn be used as a

mate for still other new reference planes, see Fig. 23; thus the total number of possible reference planes is without limit.

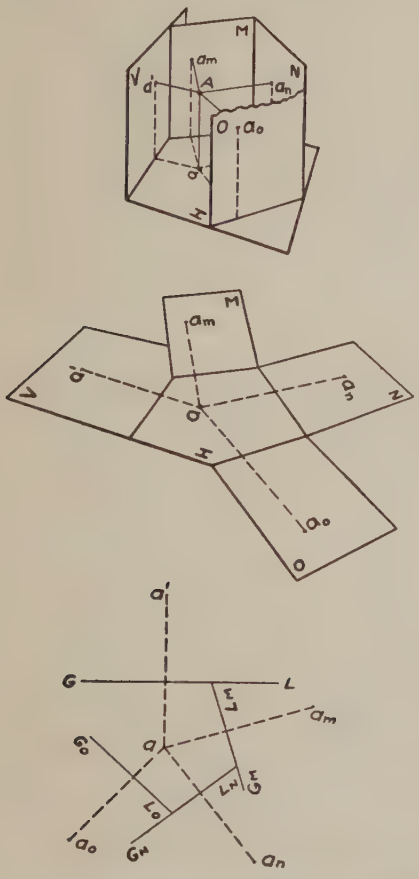


Fig. 22. Additional Reference Planes Mated with the H-Plane.

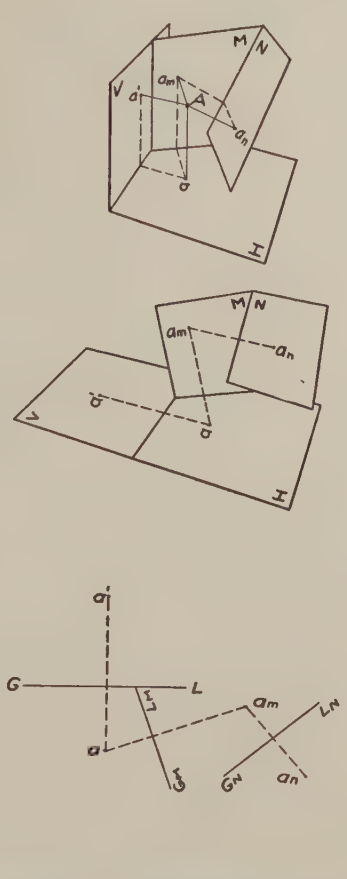


Fig. 23. A Second Additional Reference Plane Mated with a Previously Established Additional Plane.

When orthographically projecting representations upon a new reference plane, the order of projection (eye, point, plane or eye, plane, point) must agree with the order employed when projecting upon the original mate of the exist-

ing plane which serves with the additional plane in forming the new pair of reference planes. For example, if a new plane is added in the first angle using the H-plane as a mate, projection upon this new plane must be performed in the same order as for the V-plane, since the V-plane serves as the original mate of the H-plane, see Fig. 24. Application of this rule insures the maintenance of the principles which govern the type of graphic description involved; that is, it insures that every pair of reference planes employed will be used in accordance with the principles of the same type or angle of graphic description. If this rule is ignored the resulting pictures may become a hopeless jumble of first, second, third and fourth angle graphic descriptions.

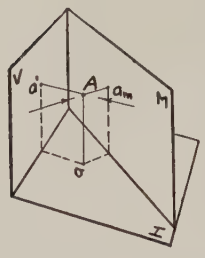


Fig. 24. The Method of Projecting upon an Additional Reference Plane — First Angle.

The folding of an additional plane is also involved with the maintenance of a particular type of graphic description. In all cases the intersection of the additional plane with the existing plane which serves as its mate must be used as a new ground line and the new plane must be folded into this existing plane, see Fig. 25. The direction of folding depends upon the primary dihedral angle in which the additional plane is employed. The following rule must always

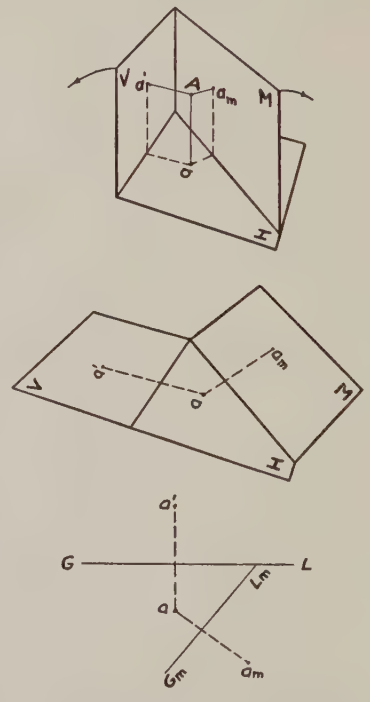


Fig. 25. The Method of Folding an Additional Reference Plane — First Angle.

be followed: The new dihedral angle, formed by the additional plane and the existing plane and containing the point or other data being described, must be opened or closed to agree with the treatment of the primary dihedral angle of the angle of description involved, see Fig. 26.

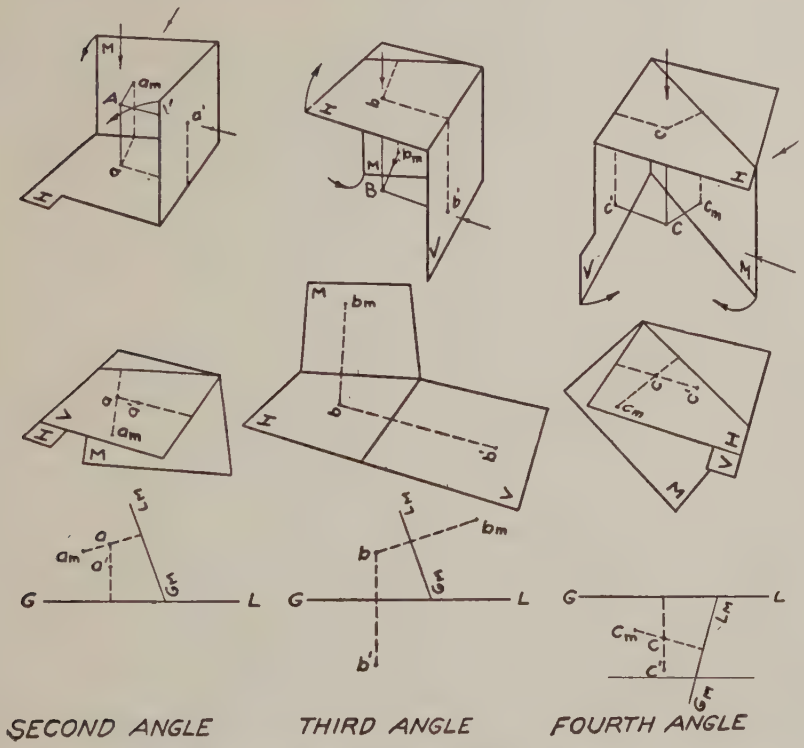
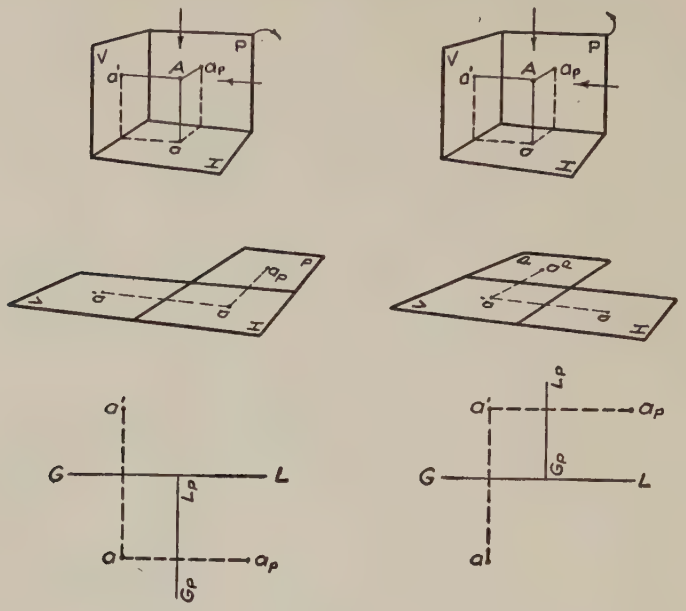


Fig. 26. Folding Additional Reference Planes for Second, Third and Fourth Angle Description.

13. Profile Planes. — If an additional reference plane is taken perpendicular to both planes of the primary pair of reference planes, it can be used with either of these planes to form a new *pair of reference planes.* Such an additional plane when used for reference purposes is called a *profile plane* and is developed for the various angles of graphic description as illustrated in Fig. 27.



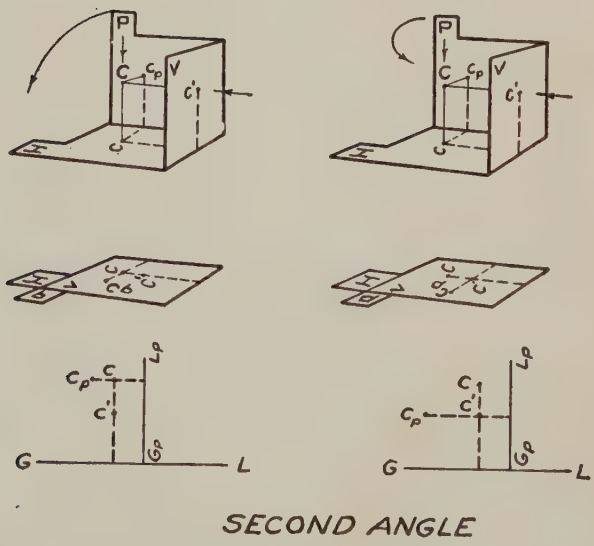


FIG. 27a. Using the Profile Reference Plane in First and Second Angles.

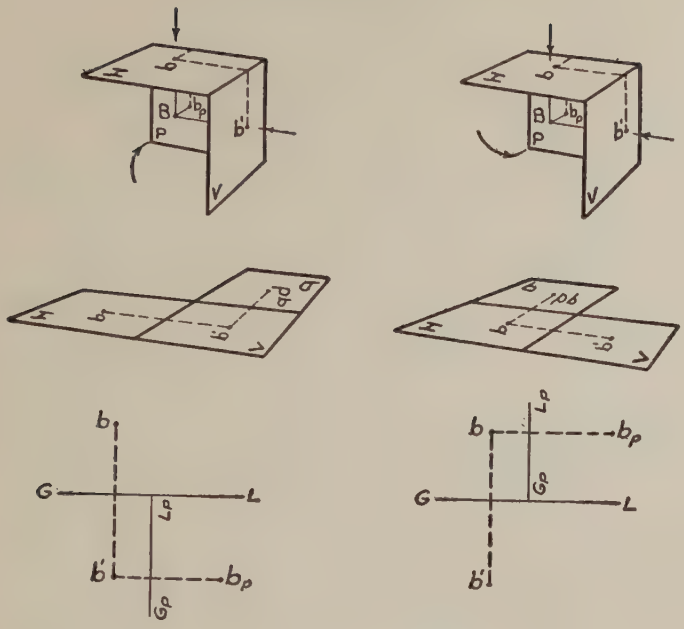


FIG. 27b. Using the Profile Reference Plane in Third and Fourth Angles.

14. Confliction of Rules. — A conflict of rules is sometimes encountered in connection with the use of an additional reference plane. This conflict occurs in connection

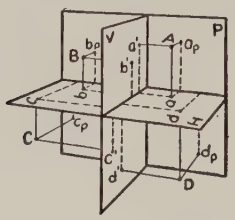


Fig. 28. Conditions which Cause a Conflict of Rules.

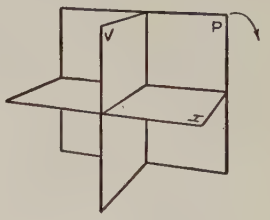


Fig. 29. Direction of Folding the Profile Plane for First and Fourth Angles when Mated with H.

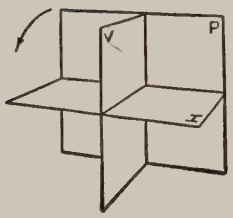


Fig. 30. Direction of Folding the Profile Plane for Second and Third Angles when Mated with H.

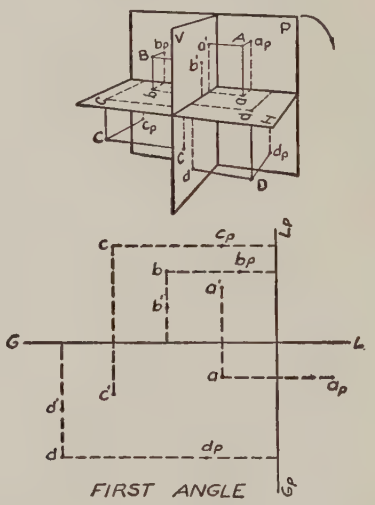


Fig. 31. A Drawing Made under Conditions which Cause a Conflict of Rules should be Labled as to Angle of Description.

with the folding of the additional reference plane when a single drawing is required to describe graphically the positions of several points located in two or more dihedral angles,

see Fig. 28. Thus if the profile plane of Fig. 28 is used with the H-plane as a mate it should according to the rules of first angle description be folded as shown in Fig. 29. This direction of folding, however, conflicts with the rules governing second and third angle description which in this instance are also involved, see Fig. 30. Confusion in such cases is avoided by folding the plane in accordance with the rules of the angle which is most favorable to the purpose of the drawing and by labeling the drawing to indicate that the rules of this particular angle were followed, see Fig. 31. In no case should the various portions of the profile plane, which serve the four angles, be separately folded in an attempt to comply with the principles of all angles.

15. Notation for Additional Reference Planes. — Additional reference planes are assigned identifying letters which are employed as subscripts when notating their ground lines. These letters are also used as subscripts when notating the representations of points projected upon the additional planes, see Figs. 21, 22, 23, 24, etc. The letter P is reserved for indicating the profile plane; other additional planes are assigned letters from the latter part of the alphabet beginning with M. Preceding letters are customarily assigned to points.

CHAPTER IV

DESCRIPTION OF GIVEN LINES, PLANES, AND SOLIDS

16. Introduction. — An understanding of the difference between the functions of graphic representation as employed in design drawing and in working drawings suggests the fact that descriptive geometry enjoys two widely differing applications.

The simpler of these two applications corresponds to the use of graphic descriptions in working drawing. This application is concerned with the graphic representation of objects for purely descriptive purposes; complete information is already available for each and every point, line, and surface described.

The more complex application is met in design drawing and is concerned with the computation of unknown sizes and shapes of objects by graphic methods. In this second application the draftsman must create points, lines and surfaces to serve the purposes of his solutions.

As may already be surmised, a practical knowledge of descriptive geometry consists of two parts: first, an understanding of the established system of graphic description and its application to the graphic representation of given points, lines and surfaces; and second, the ability to employ this applied science in planning and in executing solutions of problems involving the position of points, the position and length of lines, and the position and dimensions of surfaces.

It is the purpose of this chapter to extend the explanation of the system of graphic description from that already explained in connection with the description of given points to include the description of given lines, plane surfaces, and simple solids.

17. Straight Line Description. — A straight line is defined in terms of the descriptive unit as a row of points. A graphic description of a line obviously must consist of the descriptions of all the points which make up the line, see Fig. 32.

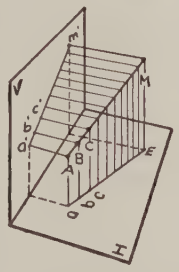


Fig. 32. The Description of a Line is made up of Descriptions of its Points.

In the practical application of this logical method of graphically describing a line, it is necessary to label only the two points which determine the extremities of the line. Thus, a line similar to the one shown in Fig. 32 would be described as illustrated in Fig. 33, and would be known as the line AB. Figs. 34, 35, and 36 illustrate respectively second, third and fourth angle descriptions of similar lines. Fig. 37 illustrates a description of a line which has its end points in different angles.

18. The Method of Expressing Information Concerning Given Points. — A convenient method of expressing the

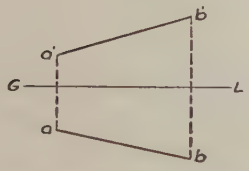


Fig. 33. A Description of a Line in First Angle.

Fig. 34. A Description of a Line in Second Angle.

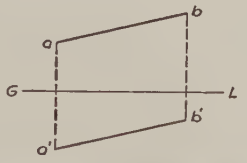


Fig. 35. A Description of a Line in Third Angle.

Fig. 36. A Description of a Line in Fourth Angle.

Fig. 37. A Description of a Line which Passes through Three Angles; First, Fourth and Third.

three positioning dimensions of a point when giving data for problems is illustrated in Fig. 38. The name of the point is

given first; then the distance from the point to a profile plane assumed to be located at the left border of the drawing sheet; next the distance from the point to the H-reference plane; and last the distance from the point to the V-reference plane. The distance from H is expressed as a positive dimension if the point is above this plane and as a negative dimension if below it. In like manner the distance from the point to the V-plane is positive if the point is in front of V and negative if in back of it. Fig. 39 illustrates the completed descriptions of a number of given points and includes the data for each in abbreviated form.

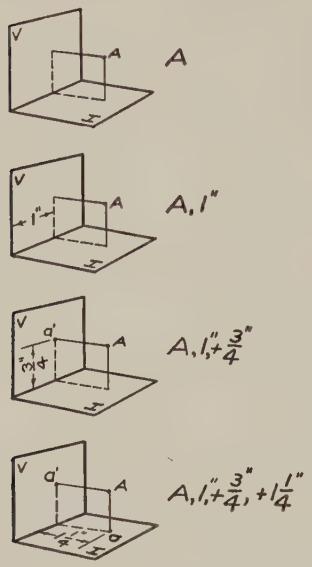


Fig. 38. The Method of Writing the Specification of a Given Point.

19. The Method of Expressing Information Concerning Given Straight Lines. — Data concerning a given straight line are expressed by specifying the names and positions of the two points which determine the extremities of the line. The method

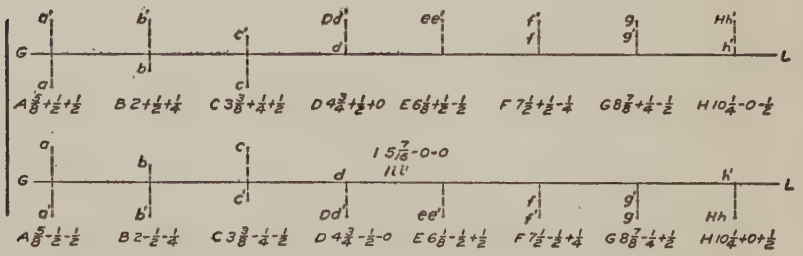


Fig. 39. Sample Descriptions of Points.

employed is a direct adaption of that established for points. The name of the line is given first; then the positions of its end points are specified, the order of specifying them being

the same as the order in which the points are mentioned when naming the line. The data required for making the drawing shown in Fig. 40 would be expressed as follows: $AB(\frac{7}{8} + 1 + \frac{3}{4})$ $(3 + 1\frac{1}{2} + 1\frac{1}{8})$. Fig. 41 illustrates the completed drawings for a variety of lines and includes the data which would be given to enable the execution of the drawings.

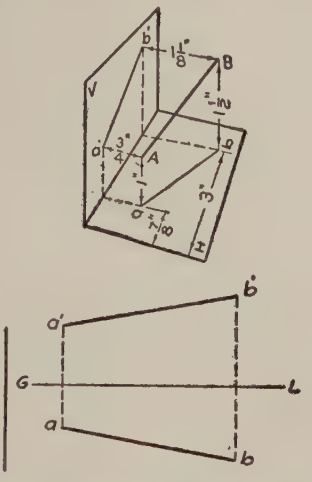


Fig. 40. The Method of Writing the Specification of a Given Line. The Specification for this Line Would Be Written AB($\frac{7}{8}$″ + 1 + $\frac{3}{4}$) (3 + 1½ + 1⅛).

20. Plane Description. — A plane in space may be defined in terms of the descriptive unit as a sheet of points. An attempt to represent a plane graphically by the methods established for points and lines, that is, by representing all the points which form the plane, shows that this scheme is not practical, see Fig. 42. However, such an attempt discloses the possibility of describing the plane by representing only those points which it has in common with the reference planes. This scheme involves the description of two lines; one being the intersection of the plane in space with the H-reference plane, and the other its intersection with V, see Fig. 43.

The established method of describing planes in space is a modification of this scheme. It consists of showing only the *actual lines* at which the plane intersects the reference planes; the *representations* of these lines being omitted. These actual lines at which the plane intersects the reference planes are called the *traces of the plane.* The intersection with the horizontal reference plane is called the *horizontal trace of the plane* in space. In like manner the intersection of the plane in space with the vertical reference plane is called the *vertical trace of the plane,* see Fig. 44.

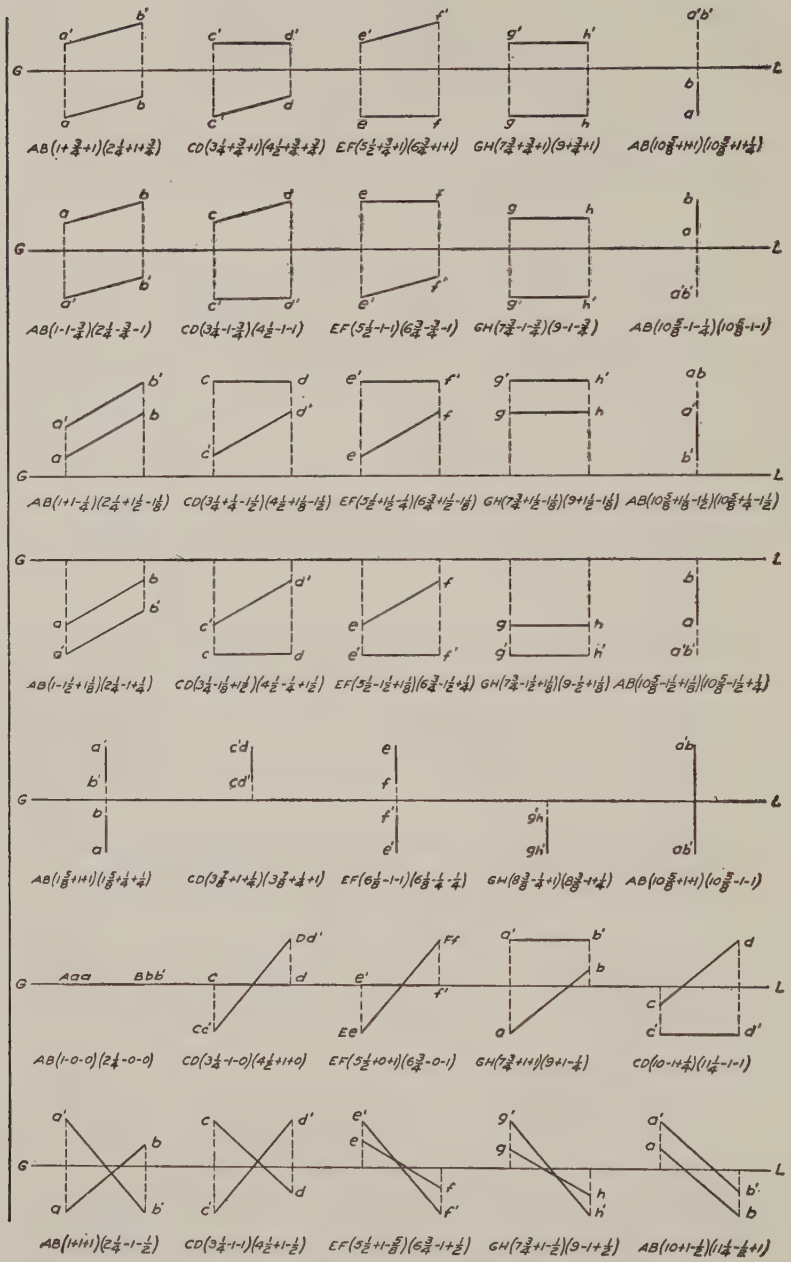

AB(1+¾+1)(2¼+1+¾) CD(3¾+¾+1)(4½+¾+¾) EF(5½+¾+1)(6¾+1+1) GH(7¾+¾+1)(9+¾+1) AB(10⅝+1+1)(10⅝+1+¼)
AB(1-1-¾)(2¼-¾-1) CD(3¼-1-¾)(4½-1-1) EF(5½-1-1)(6¾-¾-1) GH(7¾-1-¾)(9-1-¾) AB(10⅝-1-¼)(10⅝-1-1)
AB(1+1-¼)(2¼+1½-1⅛) CD(3¼+¼-1½)(4½+1⅛-1½) EF(5½+1½-¼)(6¾+1½-1⅛) GH(7¾+1½-1⅛)(9+1½-1⅛) AB(10⅝+¼-1⅛)(10⅝+¼-1½)
AB(1-1½+1⅛)(2¼-1+¼) CD(3¼-1⅛+1½)(4½-¼+1½) EF(5½-1½+1⅛)(6¾-1½+¼) GH(7¾-1½+1⅛)(9-1½+1⅛) AB(10⅝-1½+1⅛)(10⅝-1½+¼)
AB(1⅛+1+1)(1⅛+¼+¼) CD(3⅜+1+¼)(3⅜+¼+1) EF(6⅛-1-1)(6⅛-¼-¼) GH(8⅜-¼+1)(8⅜+¼+¼) AB(10⅝+1+1)(10⅝-1-1)
Aaa Bbb' Dd' Ff
AB(1-0-0)(2¼-0-0) CD(3¼-1-0)(4½+1+0) EF(5½+0+1)(6¾-0-1) GH(7¾+1+1)(9+1-¼) CD(0-1+¼)(1¼-1-1)
AB(1+1+1)(2¼-1-½) CD(3¼-1-1)(4½+1-½) EF(5½+1-⅝)(6¾-1+½) GH(7¾+1-½)(9-1+½) AB(1+1+½)(1¼-¼+1)

FIG. 41. Sample Line Descriptions.

Planes in space are notated with letters from the later part of the alphabet. The traces of a plane are notated by assigning them two letters, see Fig. 45, the first of which in-

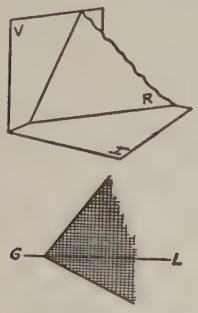


Fig. 42. The Result of an Attempt to Describe a Plane in Space by the Methods Used for Describing Points and Lines.

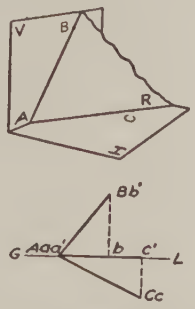


Fig. 43. The Basis of the Scheme Used for Describing Planes. This Scheme is Modified in its Practical Application.

dicates the reference plane in which the trace lies and the second indicates the plane in space which creates the trace. These letters do not apply to particular points on a trace

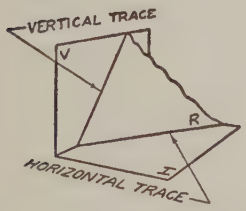


Fig. 44. The Primary Traces of a Plane.

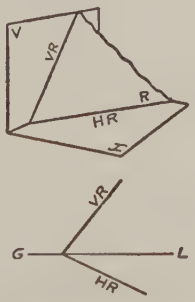


Fig. 45. The Established Method of Describing a Plane in Space.

but rather to the trace as a whole. They are shown as capital letters on drawings because they designate actual lines rather than representations of lines. Fig. 45 also illustrates a properly notated drawing which completely describes a plane *R*.

21. The Method of Expressing Information Concerning Given Planes in Space. — Since a plane is completely described by a drawing which describes its traces, the information necessary for the specification of a given plane may be expressed in terms of its traces. This information consists of the name of the plane, the angle in which it is located and either the location of two points on each of its *primary traces*[1] or the location of one point on each primary trace and the direction of each trace. The point and direction scheme requires a minimum of measurements to be made and it is therefore usually employed.

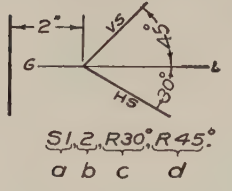


FIG. 46. The Method of Writing the Specification of a Given Plane.

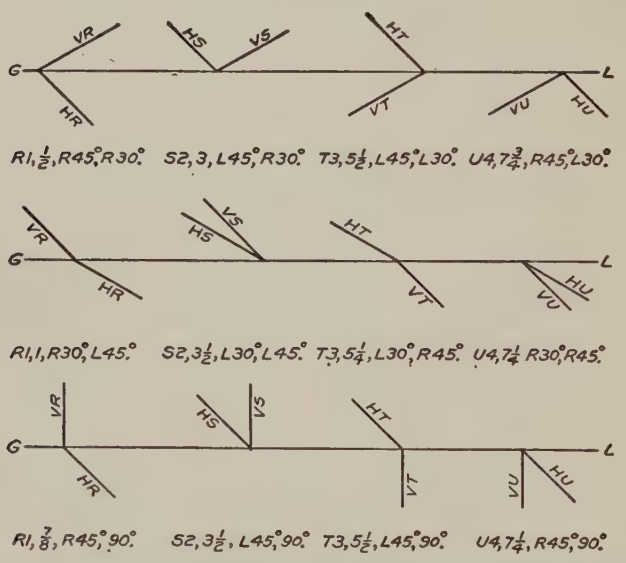


FIG. 47. Sample Descriptions of Planes which Have Intersecting Primary Traces.

When the traces of a plane meet at the ground line this scheme is used in an abbreviated form as explained in Fig. 46.

[1] The *primary traces* of a plane in space are the traces it forms with the primary reference planes.

Fig. 47 shows drawings of a number of planes and includes the data for supplying the information necessary to the execution of their graphic descriptions.

Given planes whose traces do not intersect, see Fig. 48, will be specified in full.

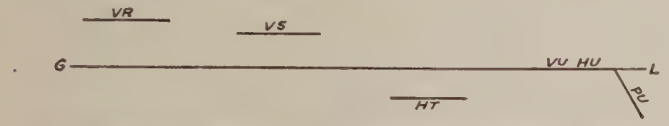


Fig. 48. Sample Descriptions of Planes whose Primary Traces Do not Intersect.

22. Description of Solids. — Solids are described by representations of their characteristic lines; only those lines are represented which are active in defining characteristic size and shape.

When the characteristic lines of a solid are straight and are clearly established in number, as for prisms and pyramids, they are represented and notated in full, see Fig. 49.

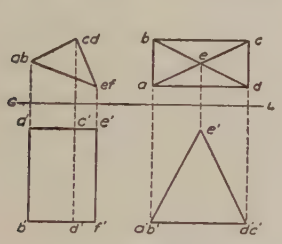


Fig. 49. Descriptions of Solids which Have Definitely Established Characteristic Lines.

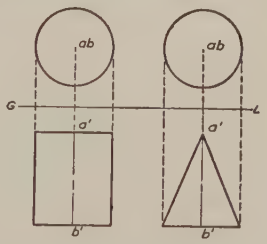


Fig. 50. Descriptions of Solids which Have Indefinite Characteristic Lines.

When the characteristic lines of a solid are not definitely established in number due to surfaces of revolution being involved, those lines of the solid which are created by the intersection of its surfaces are represented on every reference plane employed but only those representations of other lines are shown which are necessary to complete the silhouettes of the views. Notation in such cases is restricted to that required for designating axes and centers, see Fig. 50.

23. Method of Expressing Information Concerning Given Simple Solids. — The information necessary for the specification of a given solid is too involved to justify the use of special abbreviated methods such as those devised for points, lines and planes; hence it will be given in full and in a manner suited to the particular problem in hand. The following examples illustrate the methods usually employed.

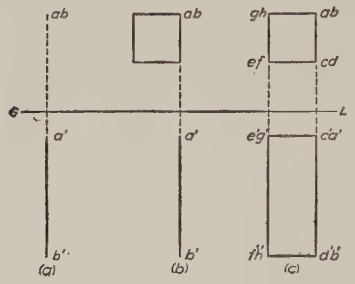


FIG. 51. The Steps of Executing the Description of a Given Prism.

(*a*) $AB(4-\frac{1}{2}-2)$ $(4-3-2)$ *is the right rear vertical edge of a* 1 *inch square prism having bases parallel to H and one face parallel to V.*

When making the drawing of this prism the first step is to represent the line AB, see Fig. 51*a*. A general knowledge of the geometric characteristics of the solid enables the H-representation of this prism to be completed, see Fig. 51*b*. Notation may then be assigned to the other characteristic lines and the V-representation completed, see Fig. 51*c*. If lines are hidden from the observer they should be shown as invisible lines after the manner of working drawing.

(*b*) $AB(3 + 2 + 1\frac{1}{2})$ $(3,0 + 1\frac{1}{2})$ *is the axis of a right circular cone with base in H and* 2 *inches in diameter.*

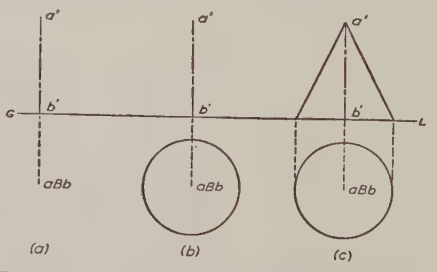


FIG. 52. The Steps of Executing the Description of a Given Cone.

When making the drawing of this cone first represent the axis AB, see Fig. 52*a*. Since the axis is not a real line its representations are shown as center lines, similar to those used in working drawing. The base is then represented and the representations of lines which determine the silhouettes of the views are added, see Fig. 52*b* and *c*.

24. Graphic Descriptions Employing Additional Reference Planes. — To meet the requirements demanded of graphic descriptions by the computative application of descriptive geometry, reference planes in addition to the primary pair must frequently be employed. The established methods of projecting points upon such planes, folding them into the drawing surface, notating, etc., have already been

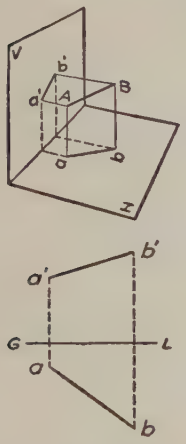


Fig. 53. The Primary Description of a Line.

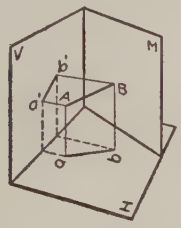


Fig. 54. Establishing an Additional Reference Plane upon which to Represent a Line.

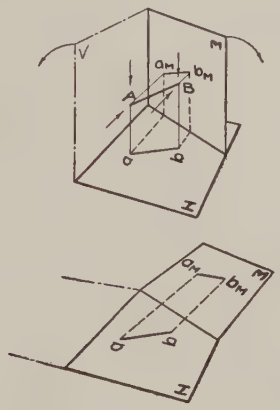


Fig. 55. The Description of a Line on a New Pair of Reference Planes.

explained. It should be fully appreciated that consideration of these factors is based upon the recognition of the new pair of reference planes formed by the additional plane and one of the primary reference planes or a previously established additional reference plane.

(*a*) *Additional Descriptions of a Line.* When it is desired to further describe a line on a drawing which already shows the primary representations, see Fig. 53, the first step is to establish an additional reference plane, see Fig. 54. The new pair of reference planes created by establishing the additional plane must be recognized before the order of

projecting upon the new plane and manner of folding it can be clearly understood, see Fig. 55. Since the representation of the line on the primary plane which serves as a mate to the additional plane is already shown on the drawing, the actual task of determining the representations borne by the new pair of reference planes is half completed at the start, see Fig. 56. The representation of the line

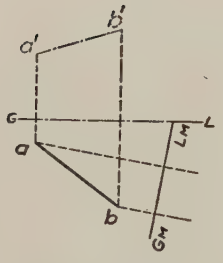


Fig. 56. The start of a Drawing which is to Show an Additional Representation of a Line.

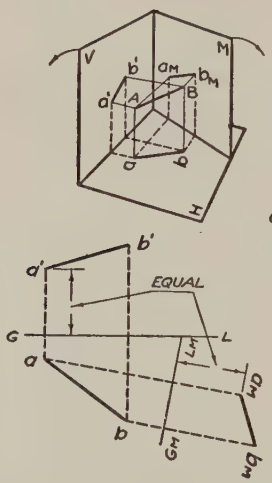


Fig. 57. Drawing the Additional Representation.

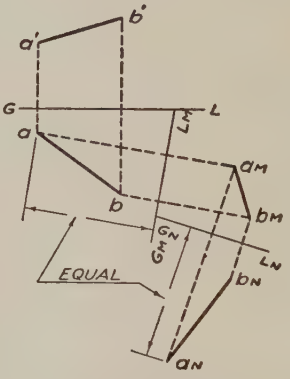


Fig. 58. Construction of a Drawing which Shows a Second Additional Description of a line. The Second Additional Reference Plane is Mated with the First to Form a Pair of Reference Planes.

on the new reference plane is, like the primary representations, fixed by the information it specifies. In the example illustrated, the new plane is used with H as a mate, hence the representation borne by it must specify the distances of all points on the line from H. This information is already specified by the original mate of H, that is, by the V-plane. The representation on the new plane is therefore drawn to

agree with that shown on the V-plane; the extremities of the new representation being located the same distance from the new G. L. as the V-representations of these points are from the primary G. L., see Fig. 57.

Fig. 58 illustrates the result of adding still another reference plane using the first additional plane as its mate.

The student should practice drawing additional descriptions of lines in all angles of description and with all possible

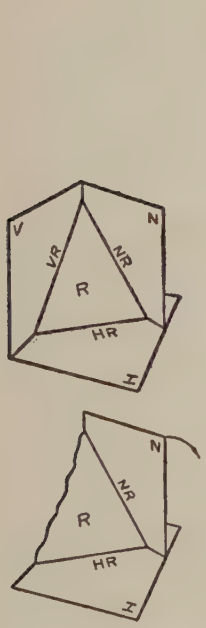


Fig. 59. Establishing an Additional Reference Plane Upon which to Describe an Oblique Plane in Space.

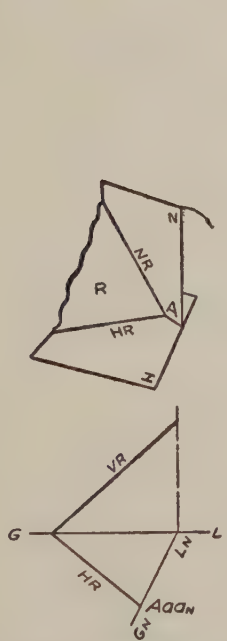


Fig. 60. Determining one Point on the Additional Trace.

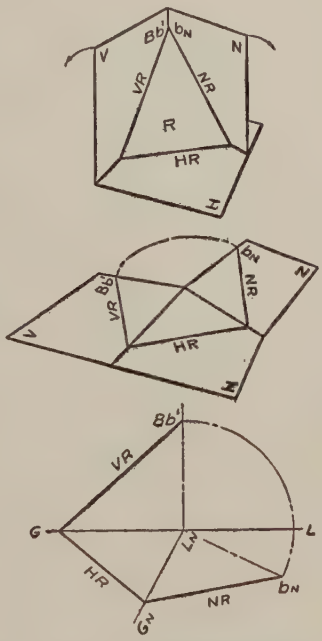


Fig. 61. Determining the Second Point on the Additional Trace.

combinations of reference planes until he is satisfied with his ability to cope with any set of conditions.

(*b*) *Additional Descriptions of a Plane.* A plane in space must obviously be described by a new pair of reference

planes in the same manner as it is described by the primary pair; that is, by its traces. As in the determination of additional descriptions of lines, the new reference plane must be established so as to use an existing reference plane as a mate and the direction of folding must be carefully noted, see Fig. 59. The trace on the new reference plane must be determined either by determining two of its points or by determining one of its points and its direction. One point on the new trace is created if the trace in the plane which serves as a mate to the new plane, intersects the new G. L., see Fig. 60. A second point is created if the trace on the new plane intersects the trace on some third reference plane which is also used as a mate to the same existing plane as the new reference plane.

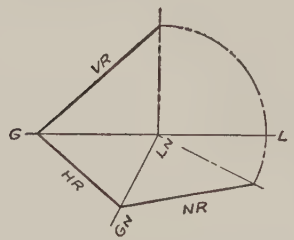


Fig. 62. A Completed Additional Description of an Oblique Plane in Space.

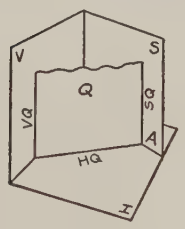


Fig. 63. Establishing an Additional Reference Plane upon which to Describe a Vertical Plane in Space.

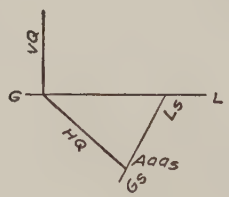


Fig. 64. Determining one Point on the Additional Trace.

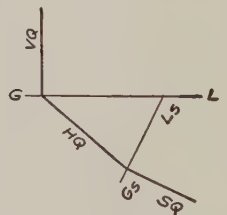


Fig. 65. The Additional Trace completed.

In the example illustrated the vertical plane is such a third reference plane, see Fig. 61. The usual construction for determining a new trace of this type is illustrated in Fig. 62.

If a plane in space is perpendicular to the existing reference plane which is used as a mate to the additional refer-

ence plane, the point and direction scheme must be employed in determining the new trace. The point on the new G. L. is

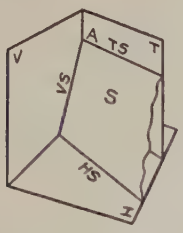


Fig. 66. The Arrangement of an Additional Reference Plane when taken Parallel to the H-Trace of a Plane in Space.

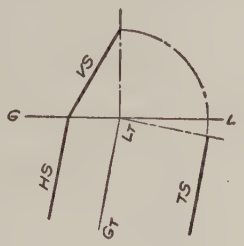


Fig. 67. The Additional Trace Determined by One Point and its Direction.

found as previously explained, see Figs. 63 and 64. The fact that the plane in space is perpendicular to the mate of the new reference plane is discovered when referring to the original mate of the existing plane. Obviously the new trace must also specify this fact and must therefore be perpendicular to the new G. L., see Fig. 65.

If an additional reference plane is taken parallel to a trace on the existing reference plane which serves as its mate, the trace on the new reference plane will be parallel to the new G. L. see Fig. 66. One point on the new trace may be determined because of the fact that it is common to this trace and to the trace of the plane in space on the original mate of the existing plane of the new pair, see Fig. 66. This point on the new trace and knowledge of its direction enables the construction of the drawing as illustrated in Fig. 67.

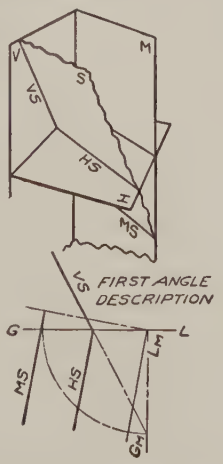


Fig. 68. Determining an Additional Trace of a Plane when the Trace Lies in an Angle other than that which Contains the Given Portion of the Plane in Space.

Fig. 68 explains the construction of a drawing which describes a plane in space and which employs an additional reference plane of such position that the trace it bears is not in the same angle as the given portion of the plane in space.

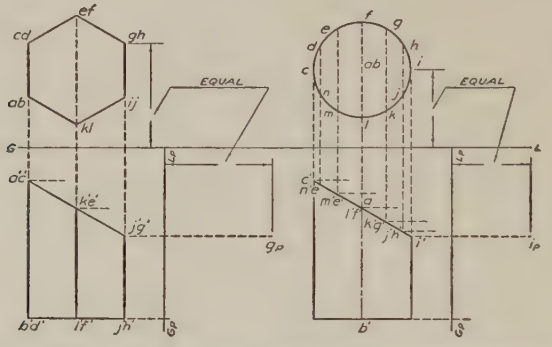


Fig. 69. Method of Determining Additional Representations of Points on Surfaces of Solids.

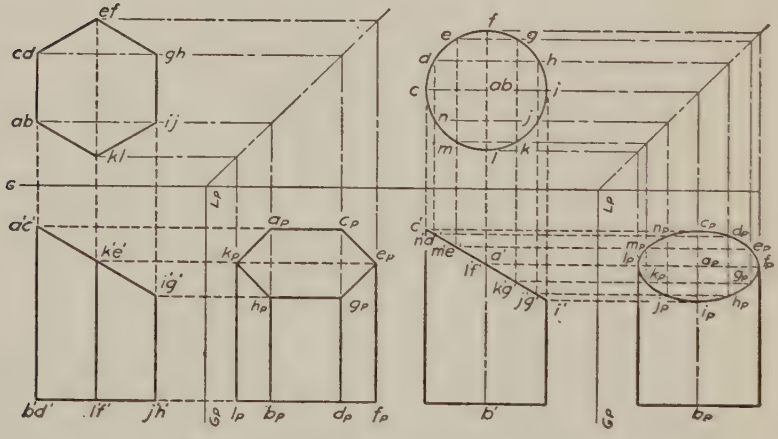


Fig. 70. Additional Descriptions of Solids.

The student should practice the additional description of planes in space which occupy all conceivable positions and should also employ all possible positions for the new reference planes.

(*c*) *Additional Description of Solids.* Representations of solids on additional reference planes are determined by determinating the new representations of their characteristic lines. This does not involve any new theory, in fact it is simply a problem of determining the new representations of points, see Fig. 69.

Fig. 70 illustrates methods of constructing additional representations of solids and graphically indicates the procedure followed.

CHAPTER V

DESCRIPTION OF REQUIRED POINTS, LINES, AND PLANES

25. Establishing a Point. — Insofar as descriptive geometry is concerned a *point* is defined as *the intersection of two lines.* A required point in space is therefore invariably

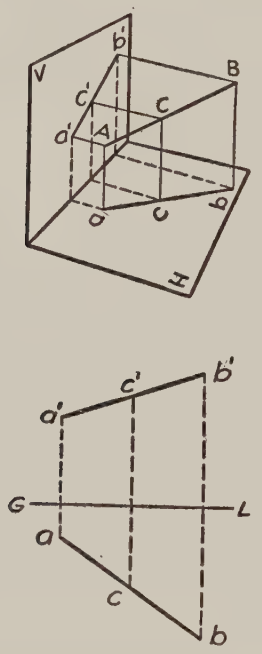


Fig. 71. If a Point Lies on a Line its Representations Must Lie on the Corresponding Representations of the Line.

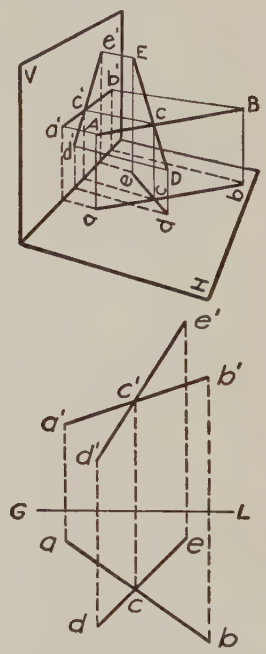
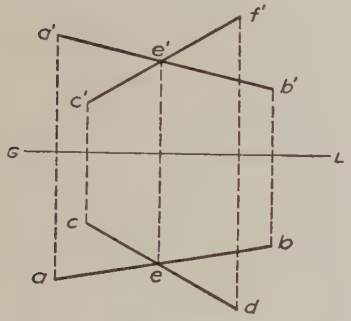


Fig. 72. If a Point Lies on Each of Two Lines its Representations Must Lie on the Corresponding Representations of Both Lines.

established by setting up a pair of lines in space which by intersecting will create it. Since the point is the unit from which lines, surfaces and solids are formed, this conception

of the procedure of establishing a point is of utmost importance.

The graphic description of a point which is common to two lines or in other words of a point created by the intersection of two lines, evidently warrants careful consider-

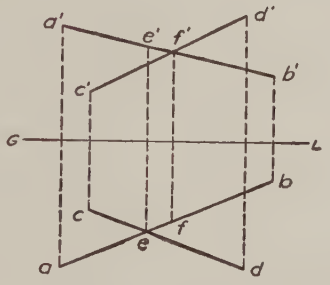


Intersecting Lines

(*a*) The V-representation of the point of intersection is located at the intersection of the V-representations of the lines.

(*b*) The H-representation of the point of intersection is located at the intersection of the H-representations of the lines.

(*c*) The intersection of the V-representations of the lines is on the same ruled projector as the intersection of the H-representations.



Non-Intersecting Lines

(*a*) The intersection of the V-representations of the lines is not on the same ruled projector as the intersection of the H-representations.

(*b*) The fact that the V-representations of the lines intersect denotes that there are two points, one on each line, which are equally distant from H.

(*c*) The fact that the H-representations intersect denotes that there are two points, one on each line, which are equally distant from V.

FIG. 73. Intersecting and Non-Intersecting Lines.

ation. Investigation reveals that if a point is on a line its representation on any and all reference planes must be on the corresponding representation of the line, see Fig. 71. Further investigation shows that if a point is on each of two lines its representation on any and all reference planes must be on the corresponding representations of both of the lines, see Fig. 72.

Many useful facts are derived from the foregoing investi-

gation, see Fig. 73, all of which find frequent application to the graphic solution of problems.

26. Piercing Points. — The point at which a line pierces a reference plane is called a *piercing point* of the line. The *horizontal piercing point* of a line is the point at which the line pierces the H-plane, see Fig. 74. Piercing points in other reference planes are similarly defined.

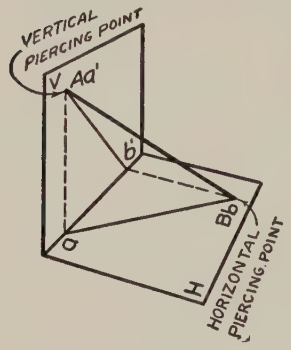


FIG. 74. The Primary Piercing Points of a Line.

The problem of determining the representations of a piercing point of a given line offers the first opportunity for the student to apply his knowledge of the principles of descriptive geometry toward the analysis and construction of a graphic solution. It is of great importance that an orderly method of analytical reasoning shall be followed when solving this initial problem because the success of future effort is largely dependent upon an early recognition of the value of systematic mental procedure. The solution of this problem is therefore given special attention.

27. Determining the Representations of a Piercing Point. — Let it be required to determine the graphic representations of the horizontal piercing point C of the given line AB, see Fig. 75.

After recognizing the characteristics of the given data (in this case it should be noted that the line AB is in first angle and is arranged relative to the reference planes as shown in Fig. 76), the first step in the procedure of the solution is to analyze the problem. This consists of first investigating in detail the requirements to be met and then devising a scheme whereby the given information concerning the problem may be applied through the medium of the science toward meeting these requirements. The procedure of reasoning outlined below seems absurdly simple and somewhat rambling but it accurately describes the nature of the mental work

which must be performed by the student in devising solutions of problems.

Q. What is wanted?

A. The vertical and horizontal representation of a point.

Q. What is a point?

A. The intersection of two lines.

Q. What two lines intersect to create the point to be represented?

A. The given line AB and some line in the H-plane, see Fig. 77, because by definition the point must be on AB and in H.

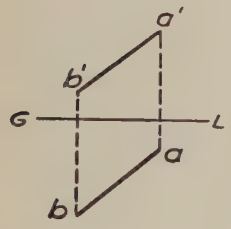


Fig. 75.

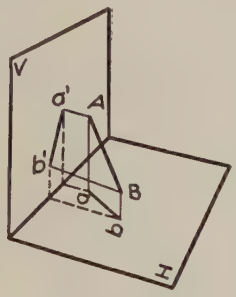


Fig. 76.

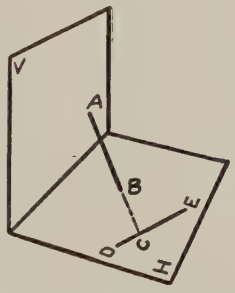


Fig. 77. The H-Piercing Point of a Line is Created by the Line Intersecting Some Second Line which Lies in the H-Plane.

Q. How can the facts now established be applied toward a solution?

A. It is known that if a point is common to two lines its representations must be shown at the intersections of the corresponding representations of the lines. Hence if the V-representations of the two lines were determined, the V-representation of the point would be formed by their intersection.

Q. The V-representation of AB is available, but where is the V-representation of the line assumed in H?

A. It is coincident with the G. L. because its points are neither above nor below H.

Q. Does the intersection of the vertical representation of AB with the G. L. form the vertical representation of the desired point? Fig. 78.

A. Yes. Because this is really the intersection of the V-representations of the two lines which create the point. Fig. 79.

Q. How can the H-representation of the point be determined?

A. It is known that it must be on a ruled projector with the V-representation and also on the H-representation of *AB*. Fig. 80.

Q. Why bother to show the representations of the line which was assumed in H, since the G. L. answers the purpose of its V-representation?

A. Now that this fact is appreciated there is no need of describing the assumed line, hence the drawing shown in Fig. 80 may be considered complete.

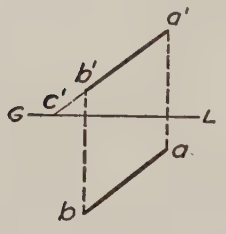


Fig. 78. Locating the V-Representation of the H-Piercing Point of a Line.

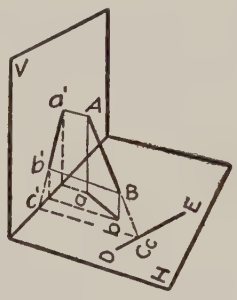


Fig. 79.

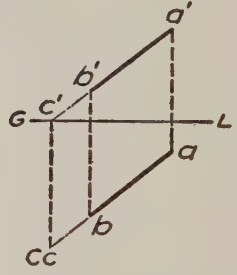


Fig. 80. The completed Description of the H-Piercing Point of a Line.

28. Determining the Representations of a Piercing Point-Second Consideration. — The above method of determining the representations of a piercing point is in accordance with the general procedure which is applied when determining the representations of *any* required point in space. A shorter method is available in the case of a piercing point because of the known relation of the point to the reference plane in which it lies. The following analysis of the problem of determining the representations of the horizontal piercing point *C* of the line *AB* shown in Fig. 75, is the result of process of reasoning similar to that given in the preceding article but in accordance with this shorter method of solution.

Analysis.

(*a*) Since by definition C is in H, its V-representation must be on the ground line.

(*b*) Since by definition C is on AB, its V-representation must be on the V-representation of AB.

(*c*) Therefore the V-representation of C must be located on the drawing where the V-representation of AB (produced if necessary) intersects the G. L.

(*d*) The H-representation of C must be on a ruled projector with its V-representation and also be on the H-representation of AB.

The above analysis not only serves to explain the shorter method of solving the piercing point problem but also illustrates the well-ordered nature of the analyses which will be in many instances a required feature of classroom solutions. This feature of the solution of a problem naturally precedes the execution of the finished drawing because it serves to compile the information essential to execution.

29. Establishing Required Lines. — A line is created by the intersection of two surfaces. Required lines, however, must be established in terms of the descriptive unit, that is, by determining the points which form them.

A required straight line may be established by either of two methods: by determining two points of the required line in space, or by determining one point of the line and its direction.

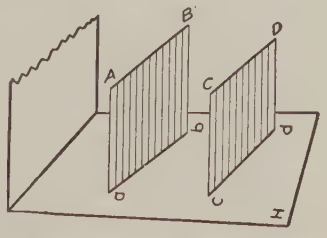


Fig. 81. If Two Lines in Space are Parallel their Corresponding Representations Are also Parallel.

Every point involved in either of the above procedures must be established by the methods previously explained, that is, by assuming a pair of lines which will intersect to create the point desired.

When *direction* is employed as a determining factor of a line, it is taken, when possible, as parallel to some existing

or purposely established second line. This method of fixing direction finds extensive application because it lends itself readily to the execution of graphic descriptions, due to the fact that *corresponding representations of parallel lines are parallel*, see Fig. 81.

30. Establishing Required Planes. — Any three points which are not on a straight line determine a plane in space. However, since planes in space are graphically described by their traces, it is necessary to employ a scheme for establishing a plane which enables the traces of the plane to be readily located.

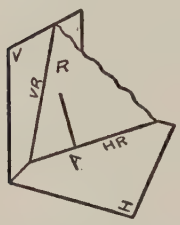


Fig. 82. A Point on the Trace of a Plane is Created by a Line of the Plane Intersecting a Reference Plane.

A *trace* of a plane, by definition, is a *line* made up of points common to a reference plane and the plane in space. A point on a trace of a plane in space is created by a line of the plane piercing a reference plane, see Fig. 82. Since two points determine a line, the horizontal piercing points of any two lines of a plane in space determine the H-trace of the plane. Likewise the vertical piercing points of these lines determine the V-trace of the plane in space, see Fig. 83.

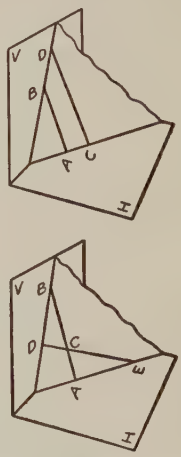


Fig. 83. A Pair of Parallel or Intersecting Lines Fix the Position of a Plane in Space.

Obviously a required plane in space may be established by first setting up a pair of lines (either parallel or intersecting) which will determine the plane in space and whose piercing points enable the traces of the plane to be located.

31. Description of a Required Plane. — Let it be required to represent a plane which shall contain three given points in space, see Fig. 84.

In accordance with the preceding discussion the first

step is to establish a pair of intersecting or parallel lines which will determine the required plane. This may be accomplished by assuming a pair of intersecting lines each of which are fixed by a different combination of the three

Fig. 84.

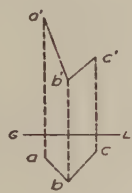


Fig. 85.

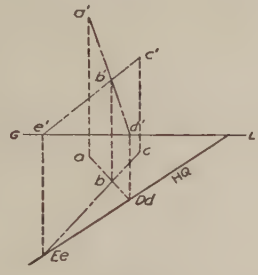


Fig. 86.

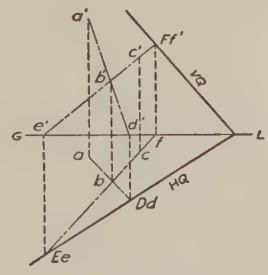


Fig. 87.

given points, see Fig. 85. Two points on the H-trace of the plane may be established by finding the H-piercing points of the two lines, see Fig. 86. Since the H- and V-traces of the plane must meet at the G. L., it is necessary to determine the V-piercing point of but one of the lines in order to fix the V-trace of the plane, see Fig. 87.

CHAPTER VI

DESCRIPTION OF SIMPLE IN-SPACE RELATIONS OF POINTS, LINES, AND PLANES

32. Introduction. — A graphic computation is simply a graphic description in which the representations of characteristic lines are augmented by the representations of those

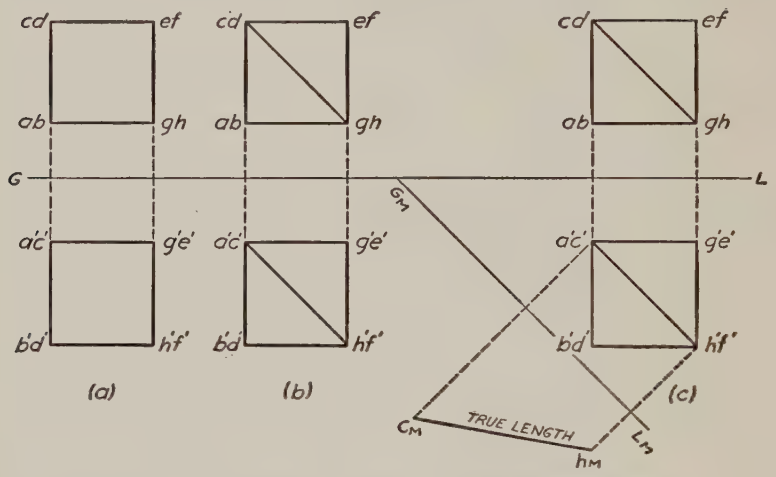


Fig. 88. An Example of a Simple Graphic Computation.

additional lines which are found necessary to define the dimensions sought.[1]

An example of a simple graphic computation is illustrated in Fig. 88. The problem is to find the length of the diagonal of a given cube. The cube is first graphically described by representations of its characteristic lines, *AB*, *BC*, etc., Fig. 88*a*. The representations of the line *CH* which defines the required dimension are then added, Fig. 88*b*. Finally, the true value of the dimension (the true

[1] Two types of dimension are employed in fixing size: *linear* and *angular*. A linear dimension is measured along a *line* between limiting *points*. Angular dimensions are measured between and in the plane of intersecting *lines*.

length of the line CH) is represented; in this case on an additional reference plane parallel to the line, Fig. 88*c*.

The lines which serve to define the dimensions sought by graphic computations are in general more difficult to conceive than the foregoing example indicates. Furthermore, the task of establishing and describing such lines, after they have been mentally recognized as necessary to a solution, requires an intimate knowledge of the methods available for this purpose.

This chapter is devoted to the discussion of the methods of describing those simple geometric arrangements of points, lines, and planes which are constantly encountered when establishing lines to define required dimensions.

33. Use of Models. — Models of the in-space relations of points, lines, and planes are of proven value in assisting with the correlation of visualization and representation. Experience gained by constructing such models is an important factor in developing the ability to visualize. The completed models serve both to illustrate the process of representation and to assist the novice in establishing beyond question the relation between actual points, lines, and surfaces in space and their graphic descriptions.

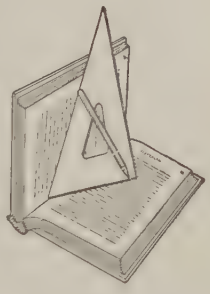


FIG. 89. A Makeshift Type of Model.

Fig. 89 illustrates a makeshift type of model which may be quickly constructed and which adequately serves many purposes.

Sheet celluloid having one surface ground so as to take pencil marks may be cut as shown in Fig. 90 and assembled to form a convenient model of the primary reference planes, see Fig. 91. Points, lines, and planes may be added in any angle and their representations marked on the celluloid planes. The pencil marks may be erased after serving their purpose.

34. Points and Lines in Planes. — Since the point is the unit by means of which lines are described and also since

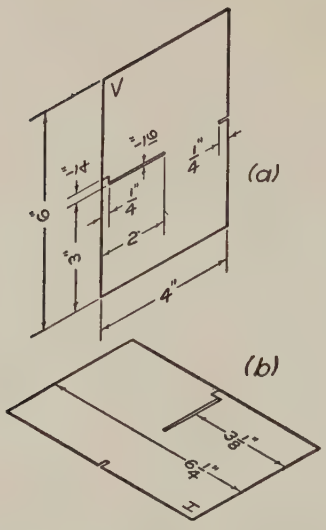


FIG. 90. Method of Cutting Sheet Celluloid for a Model of the Primary Reference Planes.

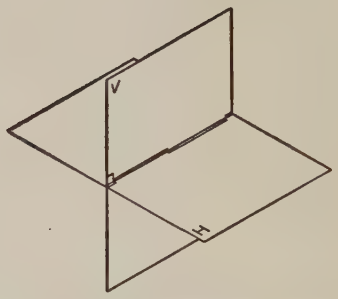


FIG. 91. The Assembled Model of the Primary Reference Planes.

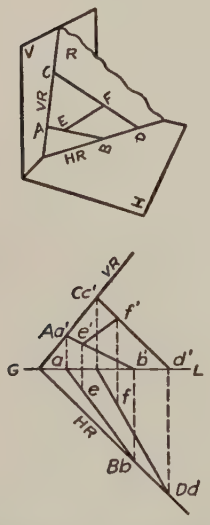


FIG. 92. If Two Points of a Line Are in a Plane, the Line Lies in the Plane.

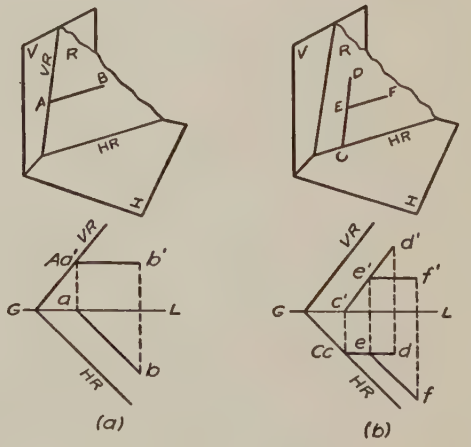


FIG. 93. If One Point of a Line Lies in a Plane and the Direction of the Line is Parallel to a Second Line which Lies in the Plane, the Line Lies in the Plane.

both points and lines must be employed to define dimensions, occasions constantly arise when it is necessary to establish points and lines in planes or in other surfaces. Methods of establishing points and lines in planes are suggested by the following geometric truths; applications of the methods are illustrated by the accompanying figures.

(*a*) *If two points of a line are in a plane the line is in that plane, see Fig.* 92.

(*b*) *If one point of a line is in a plane and the direction of the line is parallel to some line of the plane, this line is also in the plane, see Fig.* 93. The point and direction scheme of fixing a line may be applied in other ways such as: (1) *if a line is drawn through a point in a plane and parallel to a line which is parallel to the plane, the new line will be in the plane;* (2) *if a line is drawn through a point in a plane and perpendicular to a line which is perpendicular to the plane, the new line will lie in the plane.*

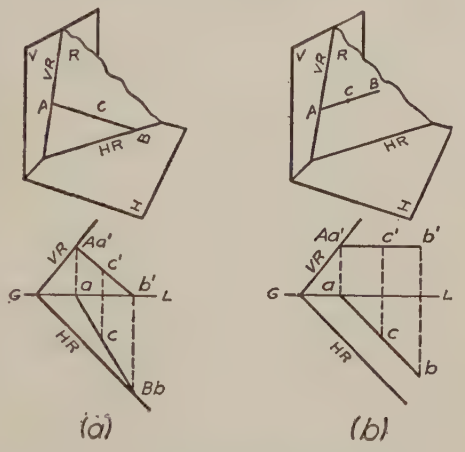


Fig. 94. If a Point Is in a Plane it must Lie on a Line of the Plane.

(*c*) *If a point is in a plane it must be on a line of that plane, see Fig.* 94.

The illustrations, Fig. 92, etc., are drawn in first angle because in-space relations may thus be shown pictorially with greater clearness. The student should familiarize himself with the use of the other angles and should also practice using additional reference planes.

35. Points and Lines in Surfaces of Solids. — Points and lines are established in other types of surfaces by the same general method that is employed for establishing them in

planes. Two principles may be set up which are applicable to all cases.

(*a*) *If a point is in a surface it is on a line of that surface, see Fig.* 95.

(*b*) *If a line is in a surface it is composed entirely of points in that surface, see Fig.* 96.

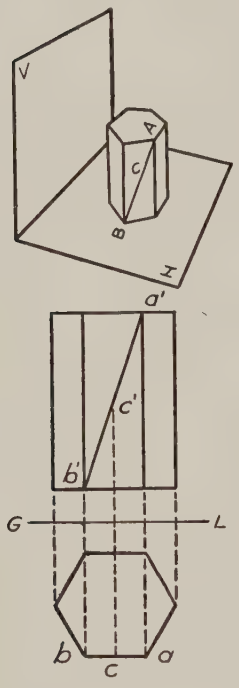


Fig. 95. If a Point Is on a Surface it must Lie on a Line of that Surface.

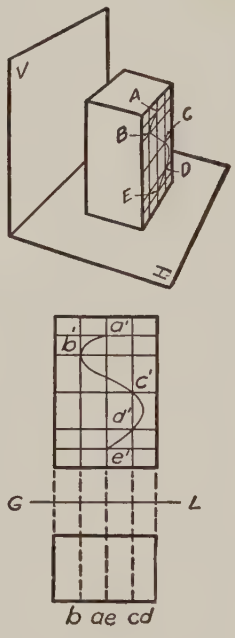


Fig. 96. If a Line Lies in a Surface it must Be Composed wholly of Points of that Surface.

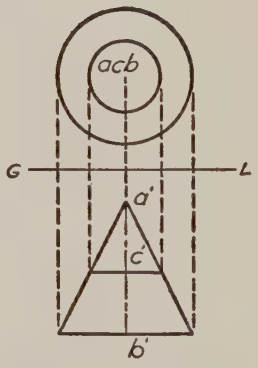


Fig. 97. A Description of a circle which Lies on a Surface of Revolution.

If the line is straight, two points or one point and its direction will determine it. If not straight, the number of points established must be adequate to enable proper representation of the line.

A circle in a surface of revolution, if parallel to a reference plane, may be readily represented without determining the representations of its individual points, provided that the position of the center and the length of its radius are known or determinable, see Fig. 97.

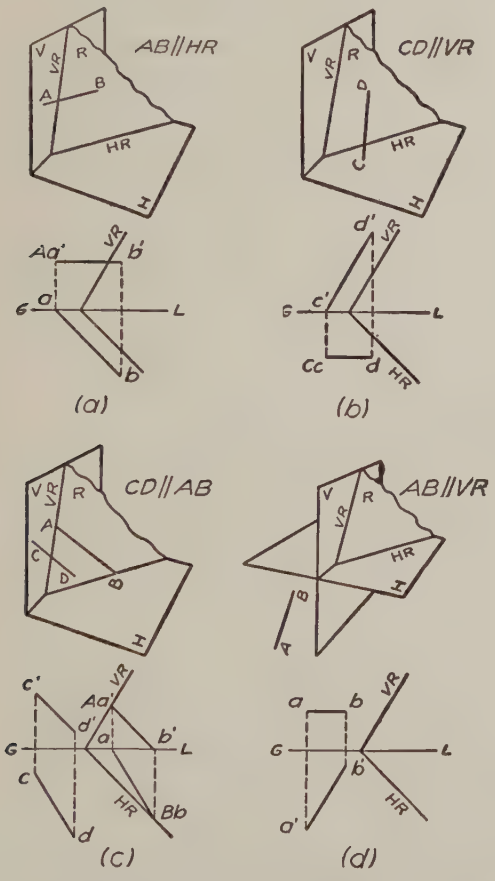


Fig. 98. If a Line Is Parallel to a Plane in Space it must Be Parallel to a Line in the Plane.

36. Lines Parallel to Planes. — A line parallel to a plane in space may be readily established and represented because of the fact that *a line parallel to a plane is parallel to some line in that plane.*

The line in the plane to which the new line is made parallel may be a trace in the plane or any other line in the plane, see Fig. 98.

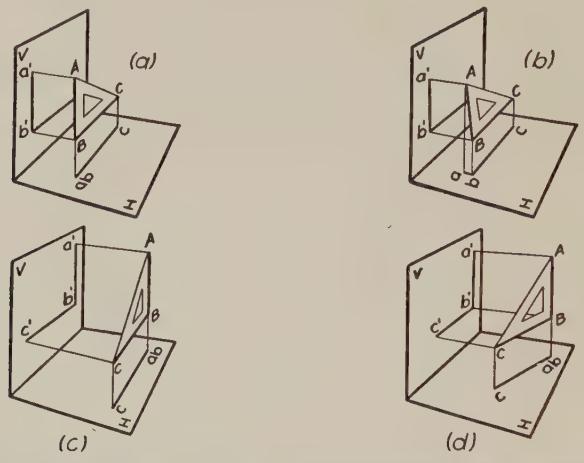


Fig. 99. If One or Both of a Pair of Perpendicular Lines Are Parallel to a Reference Plane their Representations on that Plane Are Perpendicular.

37. Perpendicular Lines. — A pair of lines which are perpendicular to each other in direction are graphically described by perpendicular representations, provided that one or both of the lines are parallel to the reference plane on which they are represented, see Fig. 99. If neither of such a pair of lines is parallel to a reference plane their representations on that plane will not be perpendicular, see Fig. 100.

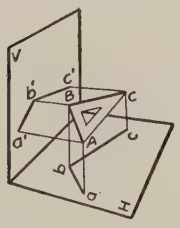


Fig. 100. If neither Member of a Pair of Perpendicular Lines Are Parallel to a Reference Plane their Representations on the Reference Plane Are not Perpendicular.

Knowledge of the geometric truths stated above enables a draftsman to establish and describe a new line perpendicular to any given line in space. For example, let the line AB, Fig. 101, be any given line in space and let it be required to add the description of a line CD which is perpendicular to AB. If CD is assumed in space parallel

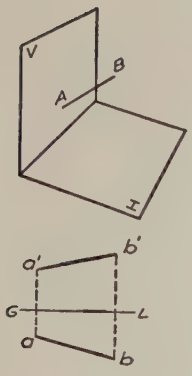


FIG. 101.

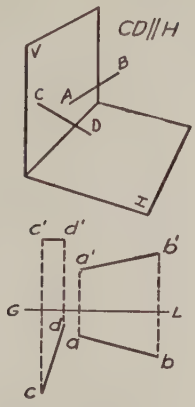


FIG. 102.

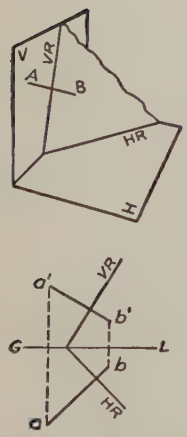


FIG. 103. If a Line is Perpendicular to a Plane it is Perpendicular to Every Line in the Plane, hence to the Traces of the Plane. Since a Trace Lies in a Reference Plane it is Perpendicular to the Corresponding Representation of a Line which is Perpendicular to the Plane.

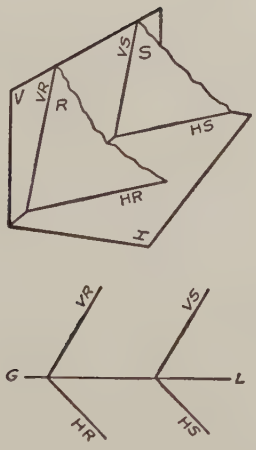


FIG. 104. The Lines Cut from Two Parallel Planes by a Third Plane Are Parallel. Hence the Corresponding Traces of Parallel Planes Are Parallel.

to H and perpendicular to AB, its H-representation will be perpendicular to the H-representation of AB, see Fig. 102. The V-representation of CD will be parallel to the G. L. since the line is parallel to H.

It should be noted that the new line may or may not be chosen to intersect the given line.

38. Lines Perpendicular to Planes. — *If a line in space is perpendicular to a plane in space it is perpendicular in direction to every line in that plane.*

Since the representations of a pair of perpendicular lines are at right angles to each other, when one of the lines is parallel to the reference plane, the H-representation of a line which is perpendicular to a plane in space must be perpendicular to the H-trace of the plane and the V-representation of the line must be perpendicular to the V-trace of the plane, see Fig. 103.

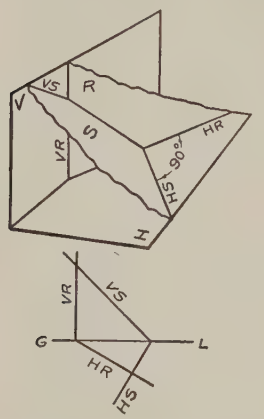


FIG. 105. The Lines Cut from a Pair of Perpendicular Planes Are Perpendicular if the Cutting Plane Is Perpendicular to One or Both of the Pair of Planes. Hence, if One of a Pair of Perpendicular Planes in Space is Perpendicular to a Reference Plane, the Traces of the Pair of Planes in that Reference Plane Are Perpendicular.

39. Parallel Planes. — *If two planes in space are parallel, their intersections with a third plane are parallel.*

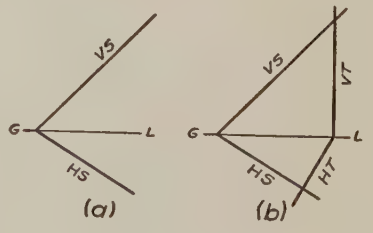


FIG. 106.

When planes are parallel in space their corresponding traces must therefore be parallel, see Fig. 104.

40. Perpendicular Planes. — *The lines cut from a pair of perpendicular planes in space by a third plane, are perpen-*

dicular to each other if the third plane is perpendicular to one or both of the pair of planes.

It is obvious therefore that if one or both of a pair of perpendicular planes in space is or are perpendicular to a reference plane, the traces of the pair of planes in that reference plane must be perpendicular, see Fig. 105. This fact enables a draftsman to establish readily and describe a new plane which is perpendicular to both a given plane and to a reference plane, see Fig. 106.

A scheme for establishing and describing any and all planes which are perpendicular to a given plane is based upon the fact *that all planes containing a line which is perpendicular to a plane are also perpendicular to that plane.* This scheme and its application is illustrated by Fig. 107. R is the given plane to which it is desired to establish perpendicular planes. The first step is to establish and describe the line AB perpendicular to the plane R. All planes containing the line AB, such as S and T are perpendicular to R.

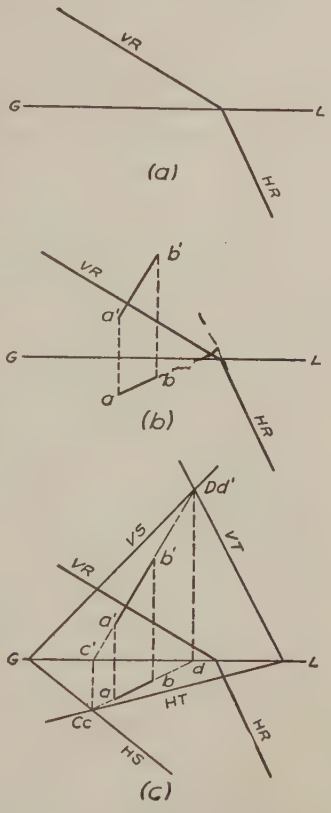


Fig. 107. All Planes which Contain a Line which is Perpendicular to a Plane Are also Perpendicular to that Plane.

CHAPTER VII

ROTATION

41. Introduction. — The true values of the dimensions sought by graphic computation are rarely found by merely establishing their defining lines and describing these lines on the primary reference planes. In order that the true lengths of lines and the true values of the angles[1] between intersecting lines may be represented, the lines must be parallel to a reference plane. When the established positions of defining lines fail to meet this requirement two methods are available by means of which the true values of the dimensions they define may be described. Either a new reference plane must be set up parallel to the lines, see the example in Art. 32, or the positions of the lines must be changed to bring them parallel to an existing reference plane. The policy of using additional reference planes seems more logical than that of changing positions as it is in strict accordance with the established system of description. However, in many cases it is possible to save considerable time and also to simplify the resulting graphic descriptions if the second method, that of changing the positions of lines, is employed. This second method consists of *rotating* the lines from their original position to the desired second position.

Rotation is not limited to the application mentioned in the above discussion; it may be used to advantage for such a variety of purposes that an intimate knowledge of its principles is essential to the efficient performance of all but the simplest of graphic solutions.

42. Basic Considerations. — Since the point is the unit in terms of which lines and surfaces are described, the prob-

[1] An exception to this rule exists in the case of perpendicular lines, see Art. 37, as but one of such lines needs to be parallel to the reference plane in order that the true angle may be represented.

lem of describing rotations must be attacked by first investigating the procedure of revolving a point. Five features are involved in this simple form of the problem, see Fig. 108:

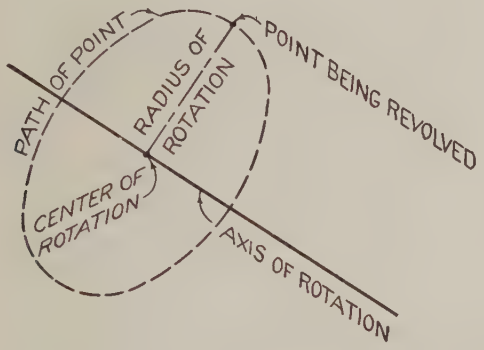


FIG. 108. The Five Features of Point Rotation.

1. *The point being revolved.*
2. A *line* about which the point is revolved, called *the axis of rotation.*
3. A *point* on the axis of rotation, called *the center of rotation.* This point is the center of the circle described by the revolving point.
4. A *line* joining the point being revolved with the center of rotation, called *the radius of rotation.*
5. *The path of the point* — the circular *line* traced by the point during its revolution. The radius of this circle is the radius of rotation; its center is the center of rotation; and its plane is perpendicular to the axis of rotation.

An investigation of the problem of describing the points and lines involved in the procedure of rotating a point discloses the following pertinent facts:

(*a*) If the axis of rotation is perpendicular to a reference plane, see Fig. 109, the plane of the circular path of the point is parallel to the reference plane. This arrangement causes all points along the axis (including the center of

rotation) to project coincidently. The path of the point is represented by a true circle and the radius by its true length. The angle between any two positions of the radius is also represented by its true value.

(*b*) If the axis of rotation is parallel to a reference plane, see Fig. 110, the plane of the circular path of the point is perpendicular to the reference plane. This arrangement results in the axis being represented by its true length and the path of the point being represented by a straight line perpendicular to the representation of the axis. All positions

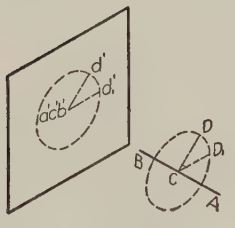


Fig. 109. Representation of a Rotation upon a Reference Plane to which the Axis Is Perpendicular.

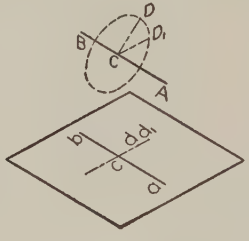


Fig. 110. Representation of a Rotation upon a Reference Plane to which the Axis Is Parallel.

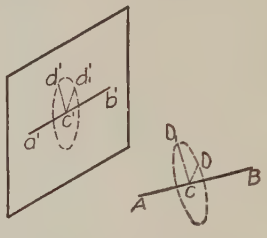


Fig. 111. Representation of a Rotation upon a Reference Plane to which the Axis Is Oblique.

of the radius of rotation project coincidently with the path of the point and its true length is represented for but two of its possible positions (when it is parallel to the reference plane). The representation of the center of rotation is located at the intersection of the representation of the axis with the coincident representations of the path and radius.

(*c*) If the axis of rotation is oblique to a reference plane, see Fig. 111, the plane of the path of the point is also oblique to the reference plane. With this arrangement the axis is not represented by its true length. The path of the point in this case is represented by an ellipse. The radius of rotation is represented by its true length in but two of its positions and the angle between any two of its positions will not be represented by its true value. Because of the

fact that the representations of two perpendicular lines are perpendicular when one of the lines is parallel to the reference plane, the representations of the radius which show its true length are perpendicular to the representation of the axis.

43. Revolution of a Point — Ideal Arrangement. — It may be readily observed from the foregoing discussion that there

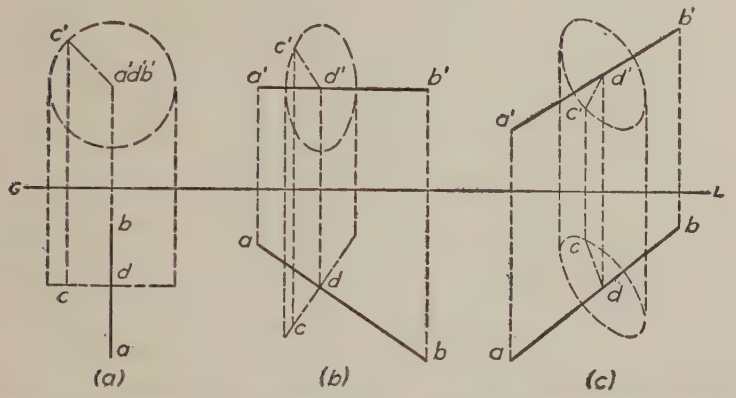


FIG. 112. Descriptions of Point Rotations for Each of the Characteristic Positions of the Axis of Rotation.

are three characteristic arrangements of the axis of rotation relative to a pair of reference planes:

1. Axis perpendicular to one reference plane and hence parallel to the other. This arrangement will hereafter be called the *ideal arrangement*, see Fig. 112*a*.
2. Axis parallel to one reference plane and not perpendicular to the other, see Fig. 112*b*.
3. Axis oblique to both reference planes, see Fig. 112*c*.

An examination of the drawings reveals that only the ideal arrangement is suited to the description of true rotational dimensions.

When the primary reference planes and the axis of rotation are not positioned in accordance with the ideal arrangement, further investigation of the problem reveals the

possibility of setting up a new pair of reference planes which will meet the conditions of ideal arrangement. If the axis is parallel to one of the primary planes, only one new reference plane is required to establish an ideal pair, see Fig. 113. If the axis is oblique to both primary planes two new reference planes must be set up before an ideal arrangement may

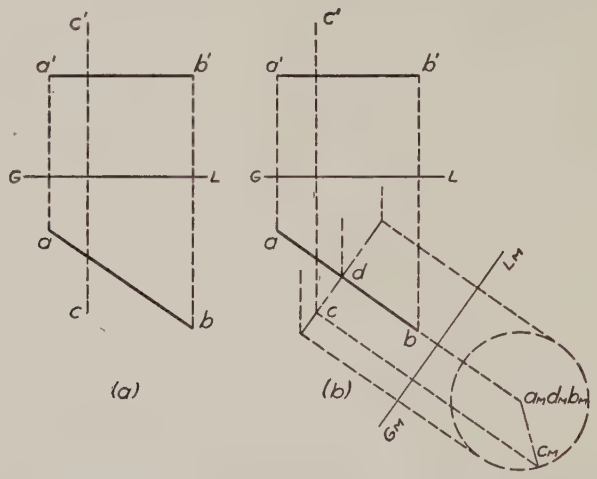


Fig. 113. Establishing an Ideal Arrangement of an Axis of Rotation by Adding a New Reference Plane.

be established, see Fig. 114. Representations of a new position of the point must first be shown on the ideal pair of reference planes; representations of this new position on the other reference planes are then obtained by treating the planes in pairs according to the established system of graphic description, see Figs. 115 and 116.

44. Shorter Procedures. — The procedure of rotating a point is rather lengthy when the rotational data are not positioned in accordance with the ideal arrangement relative to the primary reference planes. This procedure may be considerably shortened for those special cases in which the desired second position of the radius of rotation bears certain relations either to its original position or to the primary planes.

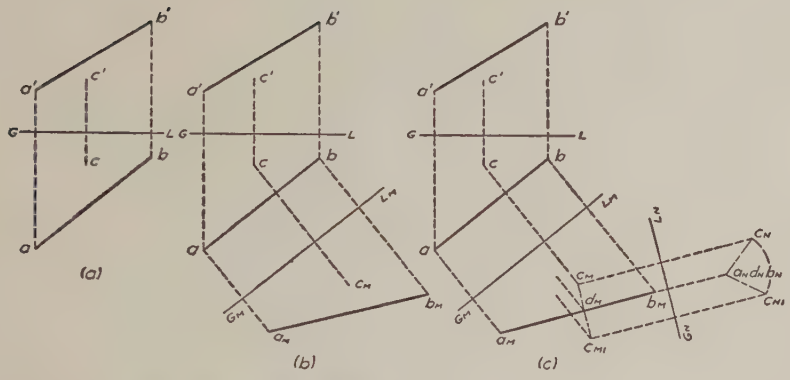


FIG. 114. When an Axis of Rotation Is Oblique to Both Primary Reference Planes, Two New Reference Planes must Be Added to Establish the Ideal Arrangement.

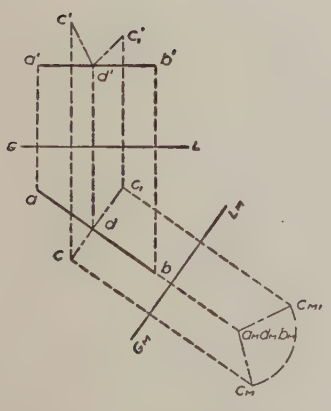


FIG. 115. A Complete Description of a Rotation when the Axis Is Parallel to a Primary Reference Plane.

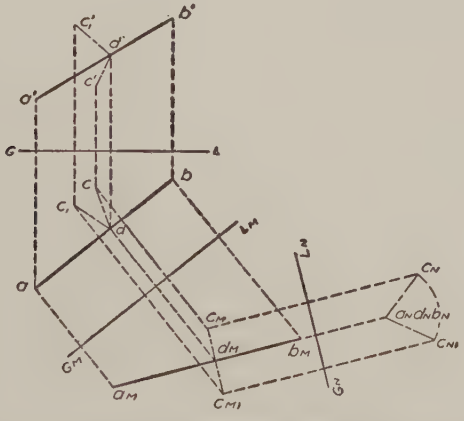


FIG. 116. A Complete Description of a Rotation when the Axis Is Oblique to Both Primary Reference Planes.

45. Special Consideration 1 — To Rotate a Point 180° when the Arrangement of Data is Non-Ideal. — In this problem the required second position of the radius is dia-

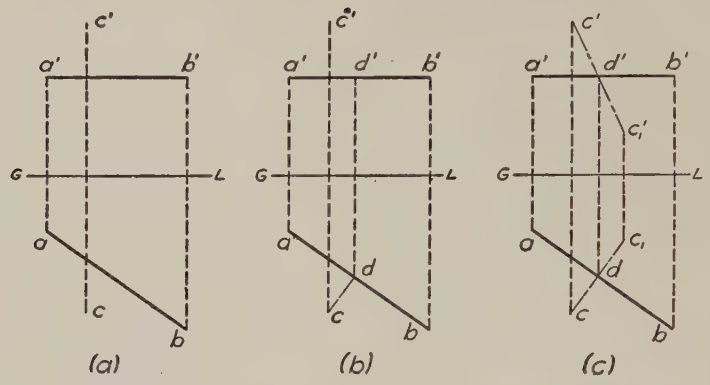


FIG. 117. To Rotate a Point 180° about an Axis which Is Parallel to a Primary Reference Plane.

metrically opposite its original position and the two positions of the radius form a diameter of the circular path of the point. Since the diameter is a straight line and the

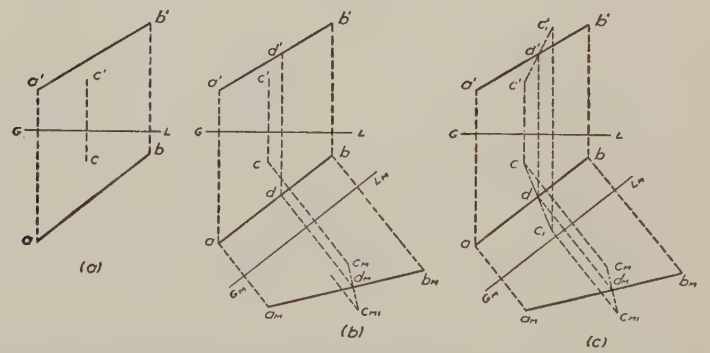


FIG. 118. To Rotate a Point 180° about an Axis which Is Oblique to Both Primary Reference Planes.

center is its mid-point, a representation of the diameter must also be a straight line and must bear the correspond-

ing representation of the center as its mid-point.[1] Knowledge of this fact enables the draftsman to describe the second position of the point, see Figs. 117 and 118. It should be observed that when the axis is oblique to both primary planes, as illustrated in Fig. 118, one new reference plane must be employed when determining the representations of the center of rotation.

46. Special Consideration 2 — To Rotate a Point to a Position in which its Radius of Rotation is Parallel to a Specified Primary Reference Plane when the Arrangement of Data is Non-Ideal. — This special rotational problem has two forms for each of the characteristic non-ideal arrangements of the rotational data.

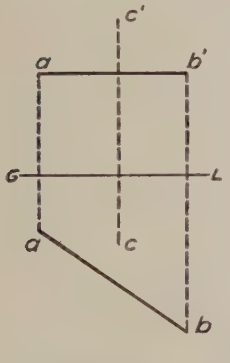
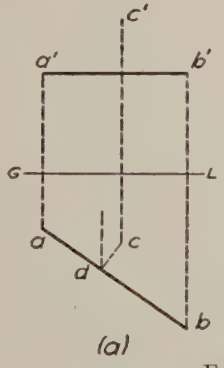


Fig. 119.



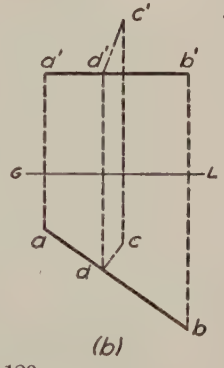


Fig. 120.

(*a*) *First Non-Ideal Arrangement — Axis Parallel to One of the Primary Planes but not Perpendicular to the Other.*

1. Let it be required to rotate the point from its given position, Fig. 119, until its radius is parallel to the same primary plane to which the axis of rotation is parallel.[2]

[1] The representation of a point on a line divides a representation of the line in the same ratio as the actual point divides the actual line.

[2] This form of the problem may be stated in several ways:

(*a*) Let it be required to rotate the point until it is at the same distance from the reference plane to which the axis is parallel as the axis is distant from that plane.

(*b*) Let it be required to rotate the point until it is as close (or as far) as possible to (or from) the reference plane to which the axis is not parallel.

It is known that the representations of any and all positions of the radius of rotation, on the reference plane to which the axis is parallel, are perpendicular to the representation of the axis on that plane. This fact enables the draftsman to locate the representations of the center of rotation and to draw the representations of the radius, see Fig. 120.

It is also known that when the radius is brought parallel to the specified reference plane its true length will be represented on that plane. If the true length of the radius were known the description could therefore be completed. Referring to the full description of this problem shown in Fig. 121, it may be seen that *the true length of the radius is equal to the hypotenuse of a right triangle one leg of which is equal to the perpendicular distance from the V-representation of the point to the V-representation of the axis and the other leg being equal to a similar dimension between the H-representations of point and axis.*[1]

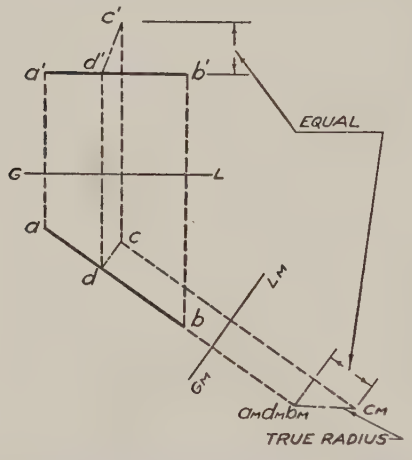


FIG. 121. The Source of the "Triangle" Scheme for Determining the True Length of a Radius of Rotation.

The triangle may be constructed at any convenient place on the paper and the true length of the radius, thus determined, may be used to fix its representation on the plane to which it is desired to be parallel, see Fig. 122. Since the radius is parallel to this reference plane its representation

[1] This rule for determining the true length of the radius of rotation can not be used unless the axis is parallel to one of the primary planes. It may be stated more concisely as follows: The true radius is equal to the hypotenuse of a right triangle whose legs are equal to the perpendicular distances between corresponding representations of the point and axis.

on the other must be parallel to the G. L., see Fig. 123. This last figure illustrates the completed description of the solution; comparison of this figure with Fig. 124 shows the saving in delineation and indicates the resulting clearness with which this method may be applied toward the solutions of complex problems involving a number of such rotations.

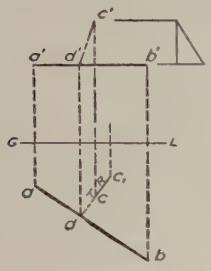


Fig. 122.

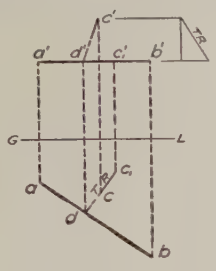


Fig. 123. To Rotate a Point until its Radius Is Parallel to the Primary Reference Plane to which the Axis Is Parallel—The Completed Shorter Description.

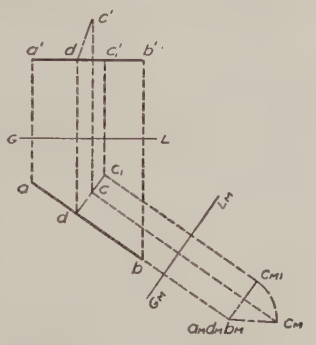


Fig. 124. To Rotate a Point until its Radius Is Parallel to the Primary Reference Plane to which the Axis Is Parallel — The Completed Full Description.

2. Let it be required to rotate the point from its given position, Fig. 119, until its radius is parallel to the reference plane to which the axis is oblique. An investigation of this required position of the radius reveals that it is then perpendicular to the reference plane to which the axis is parallel. This fact together with the knowledge gained through the previous considerations enables the draftsman to quickly arrive at the description shown in Fig. 125.

(*b*) *Second Non-Ideal Arrangement — Axis Oblique to Both Primary Reference Planes.*

1. Let it be required to rotate the point from its given position, Fig. 126, to a second position such that the radius is parallel to a specified primary plane, say the H-plane. The

first step in the solution of this problem is to describe the center of rotation. To locate the representations of the center a new reference plane must be added, see Fig. 127. The problem is then identical with problem 2 discussed under part (*a*) of this article, excepting that the primary pair of planes are replaced in the present case by the new pair formed by H and the additional reference plane. The description of the rotational data borne by the new pair of reference planes supplies all information required for locating the V-representations, see Fig. 128.

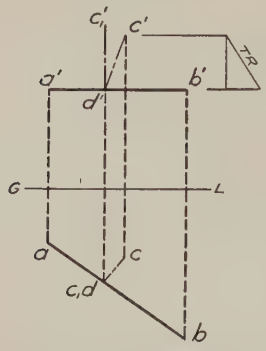


Fig. 125. To Rotate a Point about an Axis which Is Parallel to a Primary Reference Plane, until the Radius of the Point Is Parallel to the Mate of the Reference Plane — The Completed Shorter Description.

2. Let it be required to rotate a point from its given position, Fig. 126, until it is as near as possible to a specified primary plane, say the H-plane. If a new reference plane using H as its mate and parallel to the axis is added, the problem becomes similar to problem 1 discussed under part (*a*) of this article, excepting that a new pair of reference planes are employed for working purposes, see Fig. 129. The resulting description borne by the new pair of planes enables the V-representation to be readily located, see Fig. 130.

This problem may be stated in several forms, as it may be required to position the point as close as possible to either V or H or as far as possible from either V or H.

47. Rotation of Lines. — A line may be rotated to a second position by individually rotating all of the points which make up the line. If the line is straight it is necessary to rotate only its end points. In any event each point of the line traces an individual circular path and each rotates about an individual center on the axis. Obviously, to prevent distortion of the line in its new position, all points

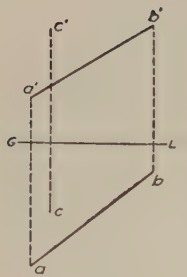


Fig. 126.

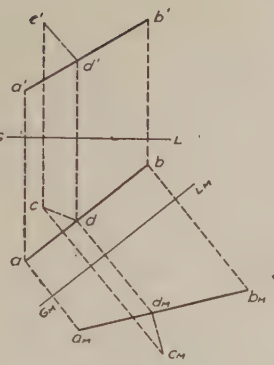


Fig. 127.

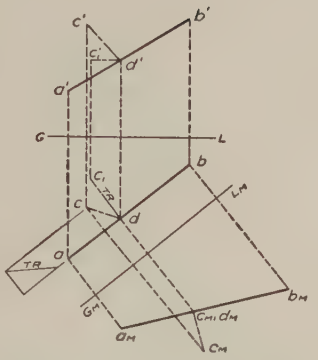


Fig. 128. To Rotate a Point about an Axis which Is Oblique to Both Primary Reference Planes, until the Radius Is Parallel to a Reference Plane — The Completed Shorter Description.

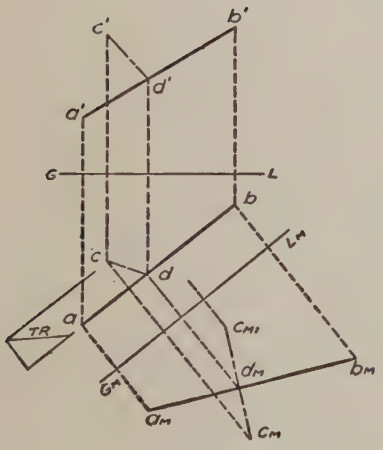


Fig. 129.

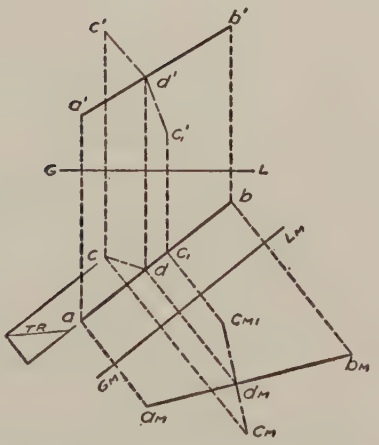


Fig. 130. To Rotate a Point about an Axis which Is Oblique to Both Primary Reference Planes, until the Point Is as near H as Possible.

must be rotated through the same number of degrees and in the same direction.

All problems of line rotation are therefore reducible to component problems of point rotation. These problems have three variations, one for each of the three characteristic positions of the axis of rotation, see Figs. 131, 132 and 133.

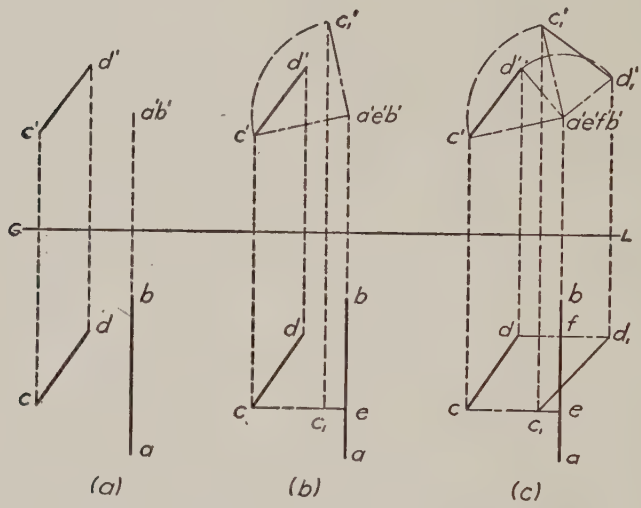


Fig. 131. To Rotate a Line — Axis in the Ideal Position.

48. The Cone Method of Determining the True Length of Lines. — A very useful application of line rotation, called *the cone method of determining the true length of a line*, enables the true length of a line to be found without using additional reference planes.

This application of rotation is based upon the fact that if the axis of a right circular cone is perpendicular to one of a pair of reference planes, two elements of the cone are parallel to the second reference plane and the true lengths of these elements are therefore represented on this plane. In Fig. 134, the axis AB is perpendicular to V: hence two elements of the cone, AC and AD, are parallel to H and are shown in their true length by their H-representations.

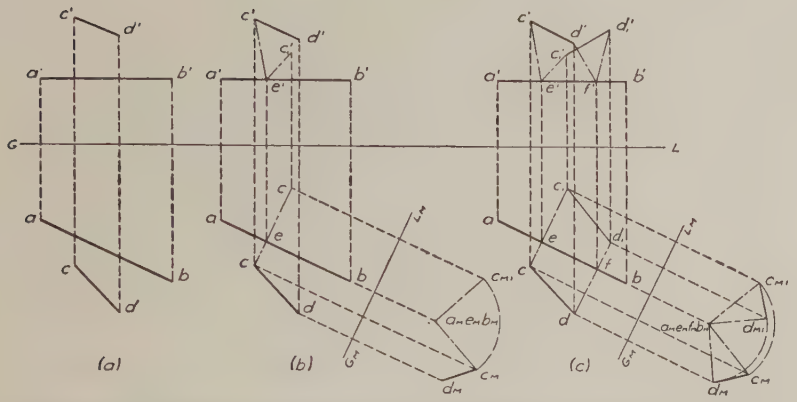


FIG. 132. To Rotate a Line about an Axis which is Oblique to One Primary Reference Plane.

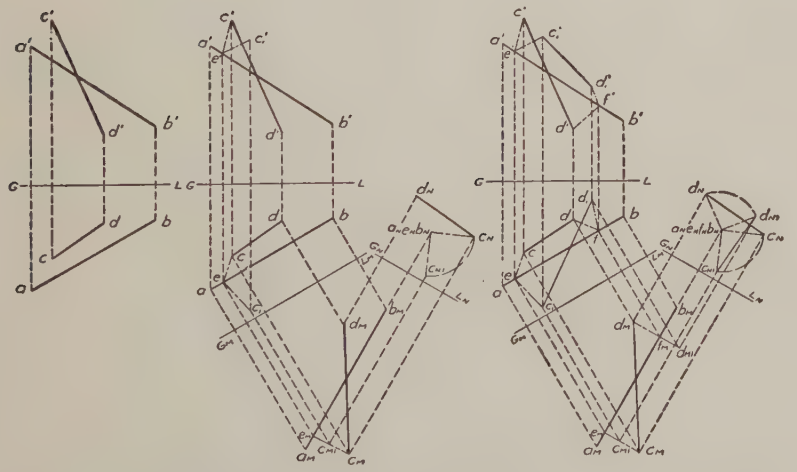


FIG. 133. To Rotate a Line about an Axis which Is Oblique to Both Primary Reference Planes.

To apply this method when determining the true length of a given line, the first step is to assume an axis of rotation through one end of the line and perpendicular to one of the reference planes. In the example, Fig. 135, the axis AC was assumed through A and perpendicular to H. If the line AB were revolved around the axis AC, a cone would be generated having A for its apex and the circle generated by B for its base circle. Since but one element of this cone is

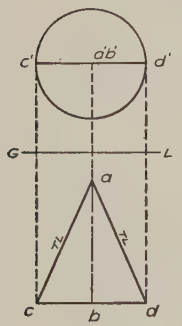


Fig. 134 If the Axis of a Cone Is Perpendicular to a Reference Plane, Two of its Elements Are Represented in their True Length on the Mate of the Reference Plane.

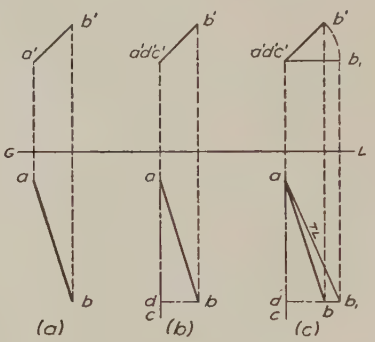


Fig. 135. Rotation of a Line about an Assumed Axis to Describe its True Length — The Full Description.

Fig. 136. The Practical Shorter Description of the Cone Method of Determining the True Length of a Line.

desired (one which is parallel to V) the line AB is actually rotated only to the position of this element.

Fig. 136 illustrates the usual description of a cone method solution. Note that representations of the axis of rotation and also the center of rotation for point B are omitted. These omissions are justified because this element of graphic description is employed so frequently that the incomplete description adequately serves its purpose and at the same time avoids congestion of lines.

Several alternative solutions of this problem are shown in Fig. 137. The solutions differ only in the choice of the

end of the given line through which the axis is assumed and in the choice of the reference plane to which the axis is

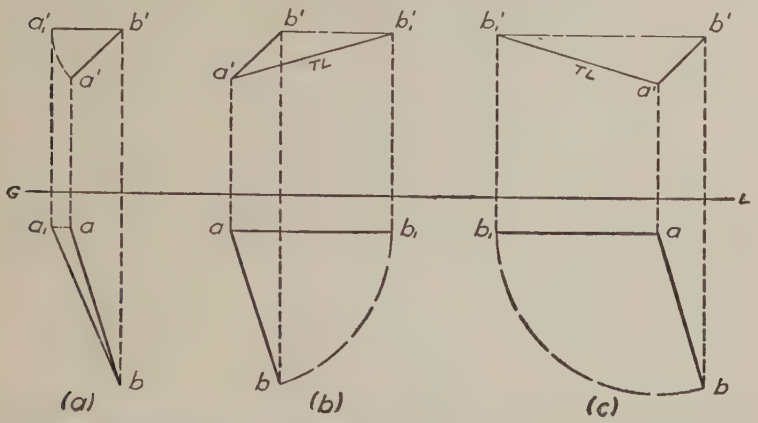


Fig. 137. Alternative Applications of the Cone Method of Determining the True Length of a Line.

assumed to be perpendicular. All of these solutions are in accordance with the principles of the cone method and all are therefore correct.

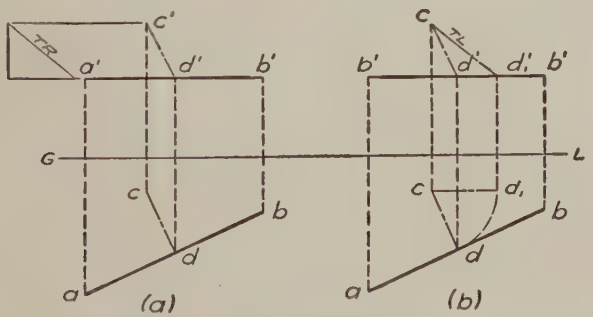


Fig. 138. Comparison of the "Triangle Method" with the "Cone Method" for Determining the True Length of a Radius of Rotation.

Fig. 138 permits a comparison of the cone method with the triangle method of determining the true length of a radius of rotation and shows that these two methods are in agreement.

49. Rotation of Planes. — Since a plane is determined by two intersecting or two parallel lines, the rotated position of a plane is fixed by rotating the lines which determine the plane. The rotated positions of these lines are in turn determined by rotating their end points as explained in Art. 48.

50. To Rotate a Plane in Space into a Reference Plane. — This problem of rotation is of importance because it offers a simple solution to the frequently arising problem of determining the true dimensions between points and lines which lie in a plane in space and also because it enables such

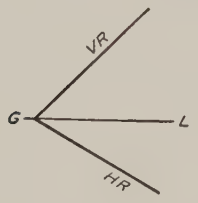


Fig. 139.

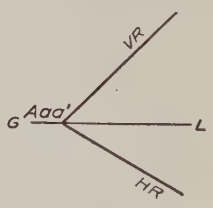


Fig. 140.

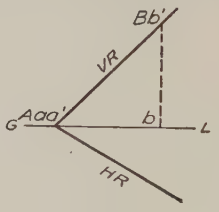


Fig. 141.

lines and points to be established in accordance with specified relations.

In its simplest form the problem consists of rotating a plane around one trace as an axis until the plane lies wholly within the reference plane which bears that trace.

Let it be required to rotate the plane R, Fig. 139, about its H-trace until it lies in the H-plane. Since the H- and V-traces determine the plane *R*, the required position of the plane may be determined by rotating these traces to their new positions. The H-trace, in this case will not be changed in position by the rotation because it serves as the axis. The revolved position of any two points of the V-trace will determine its new position. It is known that one point of the V-trace, *A*, will not be moved as it is on the axis, see Fig. 140. Any second point, *B*, on the V-trace is next selected, see Fig. 141, and rotated about the H-trace as an axis until it lies in the H-plane, see Fig. 142. The V-trace is finally added in its revolved position, Fig. 143.

The procedure as outlined above may be considerably

shortened by recognition of the fact that the portion of the V-trace between the two points A and B is shown in its true length in both its original and its rotated position. The shorter method shown in Fig. 144, is based upon this fact and is generally employed in preference to the more complete method.

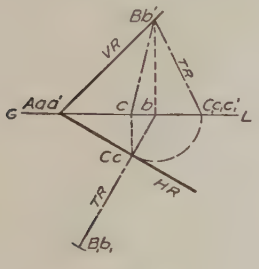


Fig. 142.

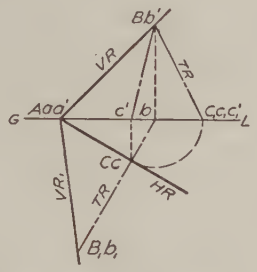


Fig. 143. To Rotate a Plane in Space into a Reference Plane — The Completed Full Description.

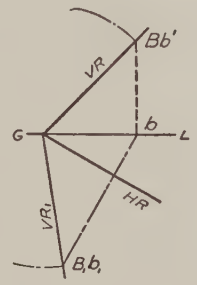


Fig. 144. To Rotate a Plane in Space into a Reference Plane — The Shorter Practical Description.

51. Rotation of Solids. — Simple rotations of plane figures and solids are executed by individually rotating their characteristic points as this determines the revolved positions of their defining lines.

When the axis of rotation is positioned in accordance with the ideal arrangement, the rather lengthy procedure of rotating each point individually may be considerably shortened by applying the following two facts:

1. The representation on the reference plane to which the axis is perpendicular is not changed in either size or shape by the rotation but is merely turned to a new position.

2. Since points rotate in planes which are perpendicular to the axis, all points stay the same distance from the reference plane to which the axis is perpendicular and their representations on the plane to which the axis is parallel therefore remain at a constant distance from the G. L. during rotation.

The application of these two facts to the execution of a drawing is illustrated by the following example. Let it be required to rotate the solid shown in Fig. 145, 30° clockwise about an axis through A and perpendicular to V. The fact that the representation on the reference plane to which the axis is perpendicular will not be changed except as to position enables the draftsman to immediately draw the V-representation of this solid in its rotated position, see Fig. 146. The fact that all points remain the same distance

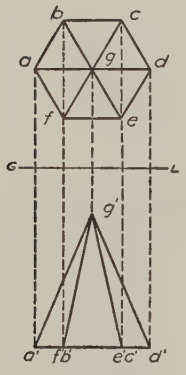


Fig. 145.

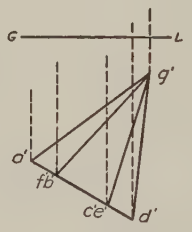


Fig. 146.

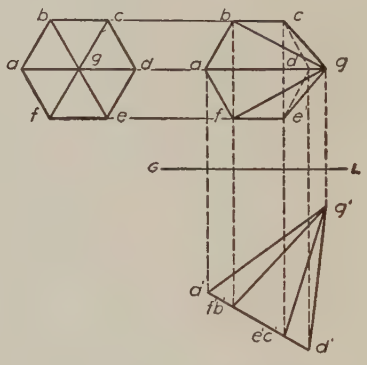


Fig. 147. To Rotate a Solid about an Axis which is Perpendicular to a Primary Reference Plane — The Completed Description.

from the plane to which the axis is perpendicular enables the H-representations of the points to be plotted and the description of the solid in its new position to be completed as shown in Fig. 147.

When the axis about which a solid is to be revolved is not positioned according to the ideal arrangement with respect to the primary planes, additional reference planes may be used to establish a new pair of reference planes which will meet the ideal condition. The description of the rotation may first be worked out with this ideal pair of planes and the representations on the original planes obtained from the information thus established.

CHAPTER VIII

DESCRIPTION OF INTERSECTIONS

52. Introduction. — The intersection of two lines is a *point* common to the two lines. The intersection of a line with a surface is a *point* common to the line and the surface. The intersection of two surfaces is a *line* made up of points common to the two surfaces.

Since the *point* is the descriptive unit and since a point is created by the intersection of two lines, the basic method of determining any intersection is founded upon a method of establishing a pair of lines which intersect to create a desired point. If the intersection itself is a point, as for a line and a surface, but one pair of intersecting lines need be established. If the intersection is a line, as for two surfaces, a pair of intersecting lines must be established for each of the points employed for fixing the line.

Evidently the entire scheme by means of which intersections are described hinges upon the possibility of establishing a pair of lines which not only intersect but which create by their intersection a point common to the intersecting data. When establishing a pair of straight lines for this purpose, two requirements must be met. First, the lines must be in a plane together or they will fail to intersect. Second, the lines must be composed wholly of points which form the intersecting members (each line being composed of points of a different member); otherwise the point created by their intersection will not be common to the intersecting members.

The method of establishing a pair of lines for the purpose of creating a point of intersection is suggested by the preceding discussion. Obviously if a plane is passed through the intersecting data it will cut a pair of lines, one from each

member, which satisfy the above requirements. Fig. 148 illustrates how a pair of lines may thus be established which will intersect to create the point at which a line in space intersects a plane in space. Fig. 149 illustrates the method of

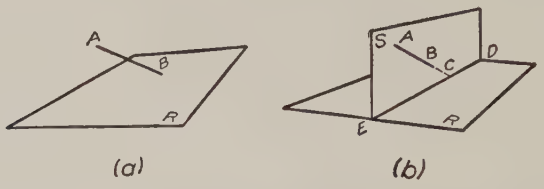


Fig. 148. The Point at which a Line in Space Intersects a Plane in Space Is Created by the Intersection of the Line with a Second Line which Lies in the Plane.

establishing a pair of lines which intersect at a point on the line along which two planes in space intersect.

Examination of the schemes illustrated in Figs. 148 and 149 reveals that in both cases it is necessary to determine

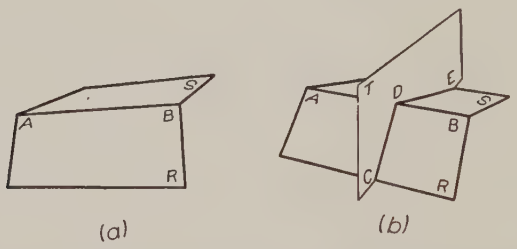


Fig. 149. A Point on the Line along which Two Surfaces Intersect Is Created by the Intersection of Two Lines, One in each of the Surfaces.

the intersection of the assumed plane with the given plane in order to establish the desired pair of intersecting lines. Upon first consideration the whole scheme seems impracticable as it appears necessary to solve a problem identical to the original before the original may be attacked. Further consideration, however, shows that this secondary problem is so vulnerable in nature that the original scheme may be

carried out. Obviously the problem of determining the intersection of two planes in space demands immediate attention.

53. Intersections of Planes — Preliminary Consideration. — The ease with which the intersection of two planes in space may be found is not due solely to the fact that the intersection is a straight line and may therefore be determined by two of its points or by one point and its direction. The fundamental reason for the simplicity of this intersection problem is the fact that planes in space are described by the actual lines (their traces) along which they intersect the reference planes. Since the corresponding traces of two intersecting planes are in the same reference plane the traces must intersect, unless they are parallel, and the point thus created must be a point on the intersection of the two planes, see Fig. 150.

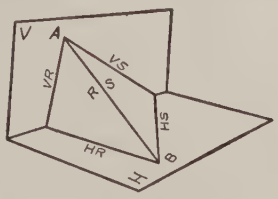


Fig. 150. The Intersection of Corresponding Traces Creates a Point on the Line along which Two Planes Intersect.

54. The Three Fundamental Problems in Plane Intersections. — All plane intersections are straight lines and are therefore determined by either finding two points of the line or by finding one point and the direction of the line. There are three simple fundamental problems in plane intersection, one of which is solved by the *two-point method* and the other two by the *point-and-direction method.* All other problems in plane intersection are solved by reducing them to these simple forms.

The first of these three fundamental problems consists of determining the intersection of two planes when their corresponding primary traces intersect within the limits of the drawing, see Fig. 151. Obviously each pair of intersecting traces create a point on the line of intersection of the planes and the line joining these two points is the line of intersection. The representations of the points may be readily located on the drawing and the representations of the line

may then be added, see Fig. 152. Several additional examples of this fundamental problem are shown in Fig. 153.

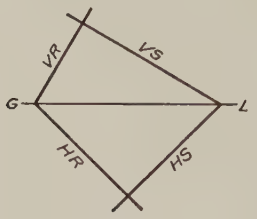


Fig. 151.

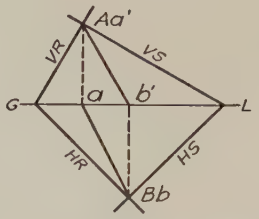


Fig. 152. The Completed Description of the Line along which Two Planes Intersect.

The second of these three fundamental problems consists of determining the intersection of two planes of such position relative to the reference planes that one pair of their

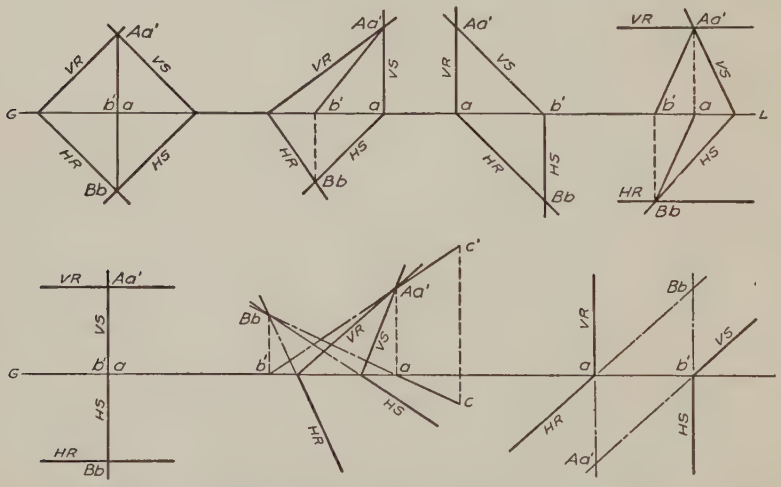


Fig. 153. Examples of Descriptions of Plane Intersections.

corresponding traces are parallel and the other pair intersect within the limits of the drawing, see Fig. 154. One point on the line of intersection is determined by the intersecting pair of traces and the direction of the line of intersection is known to be parallel to the parallel traces, see Fig.

155. The representations of the point determined by the intersecting traces may be readily located on the drawing; after this the representations of the line of intersection may

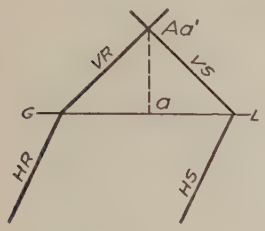
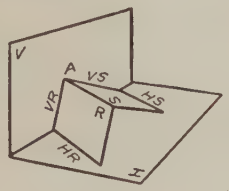


Fig. 154.



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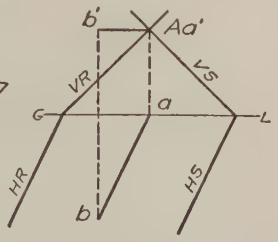


Fig. 156. The Completed Description of the Intersection of Two Planes which Have One Pair of Primary Traces Parallel.

be added, since each must be parallel to the corresponding representations of the parallel traces, see Fig. 156.

The third of the three fundamental plane intersection problems consists of determining the intersection of an oblique

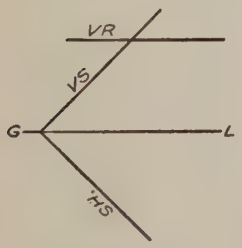


Fig. 157.

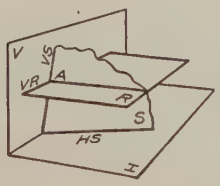


Fig. 158.

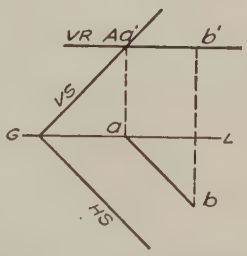


Fig. 159. The Completed Description of the Intersection of an Oblique Plane with a Plane which Is Parallel to a Primary Reference Plane.

plane in space with a plane parallel to a reference plane, see Fig. 157. Examination of this problem reveals that one point on the line of intersection may be readily determined and described because a pair of corresponding traces of the

planes intersect. It is known that if lines are cut from a pair of parallel planes by a third plane the lines are parallel. With this fact in mind it may be seen by referring to Fig.

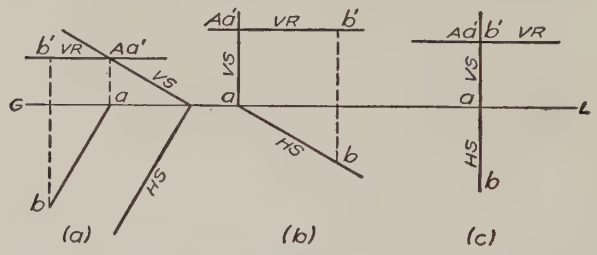


Fig. 160. Examples of Descriptions of Plane Intersections.

158 that, since R is parallel to H, the lines cut from R and H by S must be parallel; in other words the intersection of R and S is parallel to the H-trace of S. This information enables the addition of the representations of the line of intersection to the drawing, see Fig. 159. Several additional examples of this fundamental problem are illustrated in Fig. 160.

55. The General Plane Intersection Problem. — When planning the solutions of the general run of plane intersection problems the draftsman must in every case choose be-

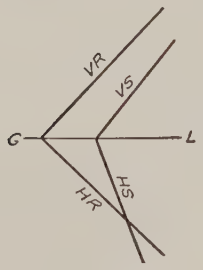


Fig. 161.

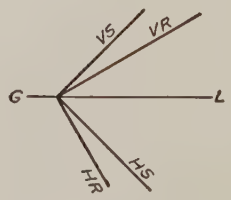


Fig. 162.

tween the two available methods of determining a line, namely, the two-point method and the point-and-direction method. The general run of problems differ from the three fundamental types explained in the preceding article only in that the given data may not directly supply the infor-

mation necessary to the immediate application of either of the two available methods of determining a line. The failure of the given data to directly supply the required information may be due to either, or to a combination, of two reasons: (1) Limitations in size of drawing sheet may make it impossible for the draftsman to employ points which are created by intersecting traces, see Fig. 161; (2) the intersecting planes may be so arranged relative to the primary reference planes that the description of the given data does not include sufficient information concerning points on the line of intersection or the direction of the intersection to enable the immediate application of either of the two methods of determining a line, see Fig. 162.

Regardless of the choice of method for determining a line of intersection, at least one point on the line of intersection must be found. If, see Fig. 163, the description of the given

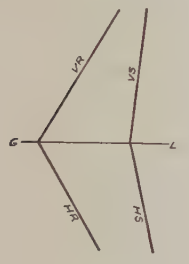


Fig. 163.

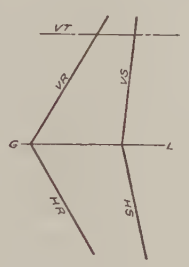


Fig. 164.

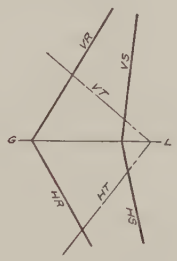


Fig. 165.

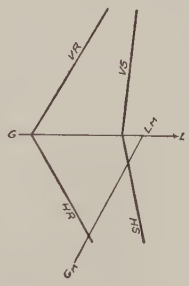


Fig. 166.

data does not directly fix such a point by including a pair of corresponding traces which intersect within the limits of the drawing, the draftsman must provide his own means of creating this initial point. Since the only method of creating a point on a line of intersection is to establish a pair of intersecting lines, one in each of the surfaces, the draftsman has no choice but to introduce a secondary intersection problem by employing a new plane which will cut the desired pair of lines. This new plane should be selected for the conveniences it offers toward the solution. If possible

it should be taken parallel to a reference plane as its intersections with the given planes may then be determined with the least effort and with the least congestion of the drawing, see Fig. 164. However, an oblique plane, see Fig. 165, or even a new reference plane, see Fig. 166, may be used.

After establishing one point on the line of intersection, see Fig. 167, the draftsman must next decide whether to

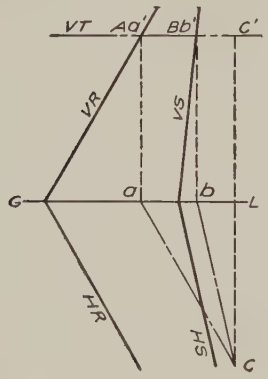


Fig. 167. A Method of Establishing a Pair of Intersecting Lines for the Purpose of Creating a Point on the Intersection of Two Planes whose Traces Fail to Meet within the Limits of the Drawing.

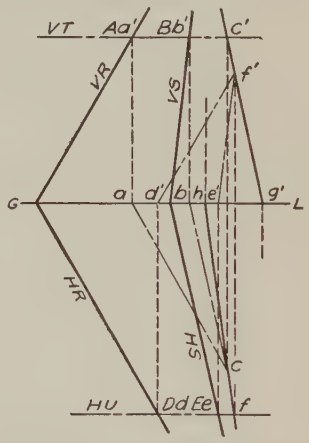


Fig. 168. A Completed Description of the Intersection of Two Planes whose Traces Fail to Meet within the Limits of the Drawing. The Two-Point Method of Determining the Desired Line Is Used in this Description.

determine a second point or to determine the direction of the line. If it is decided to determine a second point, the procedure explained above is repeated and the line of intersection is added to the drawing, see Fig. 168. If it is decided to determine the direction of the line rather than a second point, further plans must be formulated.

The fact that the direction of a line is most readily established for purposes of graphic description as being parallel to some second line has already been recognized. Such a second line may be easily determined to serve the purpose

of this solution. The scheme employed is based upon the geometric truth that parallel lines are cut from a pair of parallel planes by a third plane. Thus, if a new plane parallel to one of the given planes is assumed in such a position that its intersection with the second given plane may be readily determined, see Fig. 169, the direction of the intersection of the given planes is established by determining this secondary intersection. The representations of the

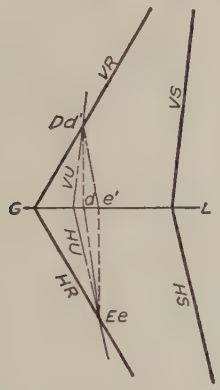


Fig. 169. The Method of Determining the Direction of the Line along which Two Planes Intersect.

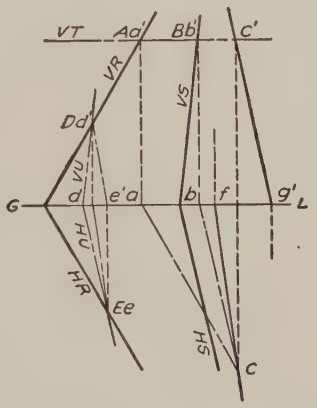


Fig. 170. A Completed Description of the Intersection of Two Planes whose Traces Fail to Meet within the Limits of the Drawing. The Point and Direction Method of Determining the Desired Line Is Used in this Description.

intersection of the given planes may now be added to the drawing because each representation of the line is known to be parallel to the corresponding representation of the secondary intersection and is also known to pass through the corresponding representation of the point which was previously determined, see Fig. 170.

56. Notes on Drafting Technique. — Students' drawings are made not only for the purpose of describing the in-space relations of points, lines and surfaces but also for the purpose of explaining the procedure by means of which these descriptions are obtained. This second purpose of the

drawings makes it necessary for the draftsman to distinguish between given, required, and assumed data and also to indicate the sequential order in which operations are performed during the procedure of the solution. The following notes on delineation practice bear directly upon these requirements of drawings.

1. Lines on the drawing which represent given data should be drawn somewhat wider than lines which represent required data, but both should be solid lines of the same color.

2. Lines of the drawing which represent extended portions of given data, and all lines which represent assumed data should be drawn as *construction lines.* Construction lines are equal in color to other lines but are of a width approximately as narrow as ruled projections. They are formed by alternate long and short dashes. The short dashes are drawn from $\frac{1}{16}$ to $\frac{1}{8}$ inch long and are separated from the longer dashes by gaps of $\frac{1}{32}$ to $\frac{1}{16}$ inch. The length of the longer dashes depends upon the nature of the drawing; it is best practice to show only a sufficient number of short dashes to indicate clearly the type of line intended.

3. The sequence of the notation assigned to points should indicate the order of determining the points during the procedure of the solution and all the points employed for the solution should be notated.

57. Illustrative Examples of Plane Intersections.

Example 1. Let it be required to describe the intersection of the two planes shown in Fig. 171.

A preliminary examination of the given data reveals the fact that one point on the line of intersection may be readily determined because the H-traces of the planes intersect within the limits of the drawing, see Fig. 172. A choice must now be made between the two-point and the point-and-direction methods of determining the line.

If the two-point method is selected, the next step is to decide upon a means of establishing the pair of intersecting lines for the purpose of creating the second point. Of course a plane must be employed to cut the pair of lines, one from each of the given

planes. A plane in space or a new reference plane may be established for this purpose. The drawing which results from employing a plane in space parallel to H is shown in Fig. 173. Fig. 174

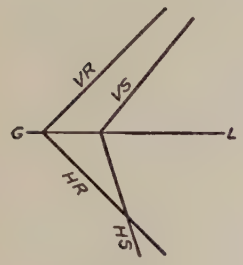


Fig. 171.

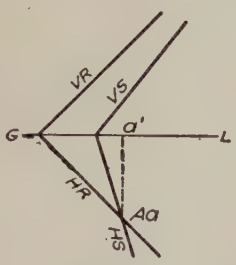


Fig. 172.

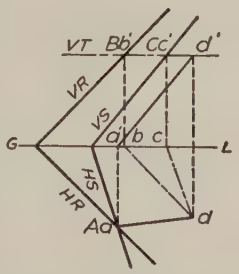


Fig. 173. A Preferred Completed Description of the Intersection of Two Planes when One Pair of Traces Fail to Meet within the Limits of the Drawing.

illustrates a drawing in which the auxiliary plane is taken oblique to both V and H. Fig. 175 illustrates the use of a new reference plane for determining the second point on the line of intersection. All of these planes are correctly used, but the scheme illustrated

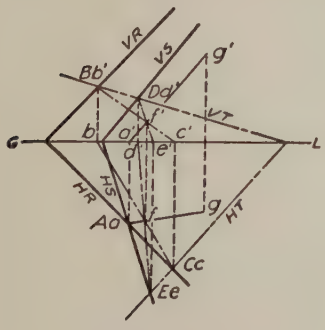


Fig. 174.

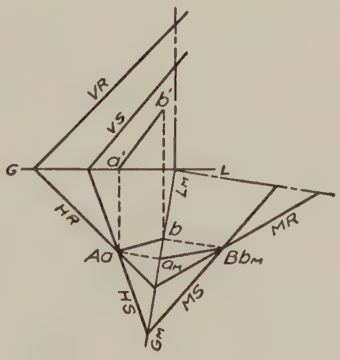


Fig. 175.

in Fig. 173 is usually preferred because less notation is required. For sake of accuracy the two points which determine a line should be established with a maximum of distance between them. This requirement often dictates the choice of the auxiliary plane.

If, instead of determining the line by the two-point method, it is decided to use the point-and-direction scheme, the draftsman must establish a line of the required direction. This is accomplished, as previously explained, by setting up an auxiliary plane parallel to one of the given planes. The intersection of this auxiliary plane with the second given plane is parallel to the desired line of intersection. Fig. 176 illustrates the drawing which results from this method of solving the problem.

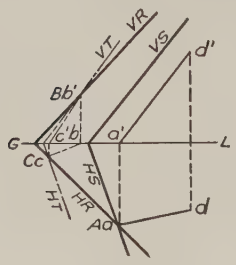


Fig. 176.

Example 2. Let it be required to determine the line of intersection of the two planes shown in Fig. 177. An examination of the given data shows that both pairs of corresponding traces of these planes intersect but that they intersect at the same point. Obviously either a second point or the direction of the line of intersection must be determined in addition to this point in order to solve the problem.

If the two-point method is adopted for the solution, the second point may be determined by using an auxiliary plane as shown in Fig. 178, or a new reference plane may be employed for this purpose, see Fig. 179. The draftsman is not limited in his choice to the type of auxiliary plane shown in Fig. 178 but may select any plane that cuts a line from each of the given planes; however, as previously noted, an auxiliary plane parallel to a reference plane

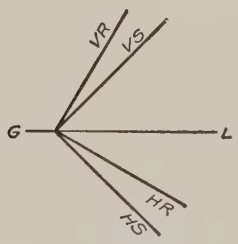


Fig. 177.

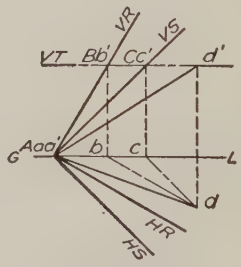


Fig. 178. A Completed Description, by the Two-Point Method, of the Intersection of Two Planes whose Primary Traces Intersect at a Common Point.

offers advantages in executing the drawing. In Fig. 179 a profile plane is used as the new reference plane. It should be fully understood that any new reference plane might be used for this purpose,

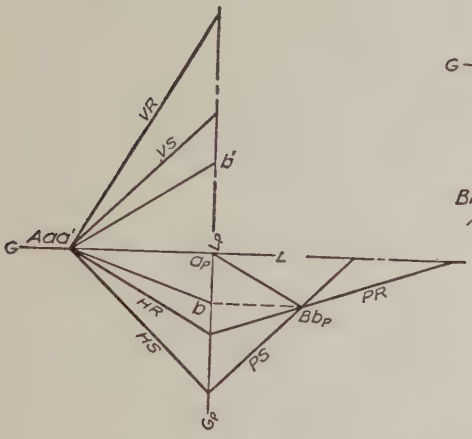


Fig. 179.

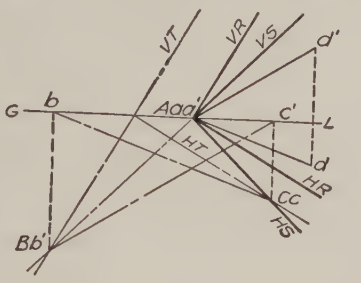


Fig. 180.

providing it cuts traces from the given planes which intersect within the limits of the drawing.

If the point-and-direction method is adopted for the solution of the problem, the direction of the line of intersection is established as parallel to the intersection of one of the given planes with an assumed auxiliary plane taken parallel to the other given plane. Fig. 180 illustrates the drawing resulting from this method of solving the problem.

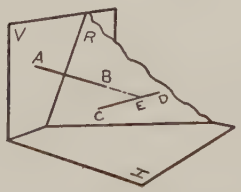


Fig. 181. The Point at which a Line in Space Intersects a Plane in Space Is Created by the Line Intersecting Some Line in the Plane.

58. Intersection of a Line with a Plane. — The intersection of a line in space with a plane in space is a *point*. Like all other points, this point is considered to be created by the intersection of two lines. Obviously the two lines are: (1) the given line in space; (2) a line in the given plane, see Fig. 181.

In order that a line in the given plane shall intersect the

given line in space the two lines must lie in the same plane. Hence, the method of establishing the second line is to cut it from the given plane by an auxiliary plane which passes through the given line.

Fig. 182 illustrates the given data for a problem of this nature. A solution of the problem is shown in Fig. 183. In this solution an oblique auxiliary plane is passed through the given line and the intersection of this auxiliary plane with the given plane is determined. The point at which the

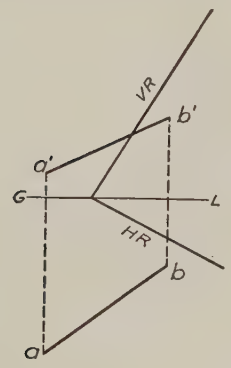


Fig. 182.

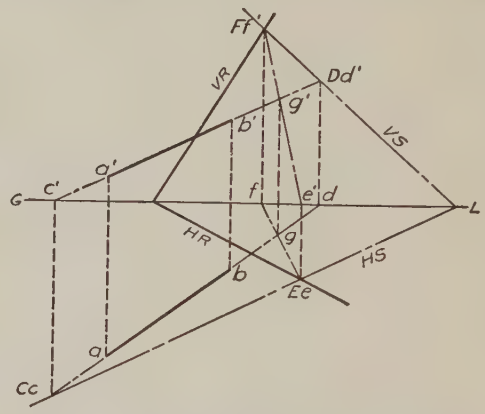


Fig. 183. A Completed Description of the Point at which a Line in Space Intersects a Plane in Space. See Fig. 184.

given line intersects this line of intersection is the point at which the given line intersects the given plane.

Fig. 184 illustrates a solution of the above problem in which the auxiliary plane is chosen perpendicular to a reference plane. The greater ease of establishing an auxiliary plane of this nature is apparent and amply justifies the usual practice of employing such planes.

59. Representation of a Line in Space Upon a Plane in Space. — The representation of a line in space upon a plane in space is created, as are all representations, by the intersections of projecting rays from the points of the line with the plane. This representation of a line is graphically

described in the same manner as the line itself, that is, by representing it upon the established reference planes, see Fig. 185. Subscripts denoting the plane in space are employed to distinguish the description of the representation from the description of the line itself, see Fig. 185. These subscripts are shown as lower case letters to emphasize the fact that they refer to representations of a representation rather than to original representations; upper case subscripts are employed only when notating original representations upon additional reference planes.

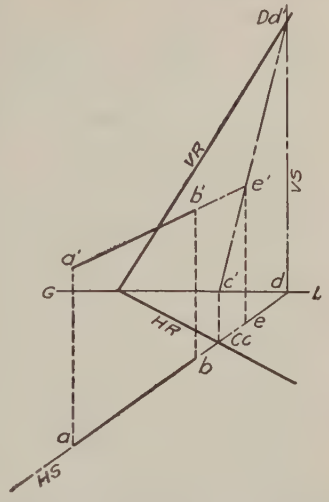


Fig. 184. A Completed Description of the Point at which a Line in Space Intersects a Plane in Space. The Auxiliary Plane Used in this Preferred Description Is Taken Perpendicular to a Primary Reference Plane.

The task of describing the representation of a line in space upon a plane in space other than a reference plane is far from difficult. The representation is created by the intersection of projecting rays with the plane, hence it is necessary simply to establish these rays as lines and to determine and describe the points at which they intersect the plane. The problem is, therefore, simply a form of the general problem of determining the point at which a line intersects a plane.

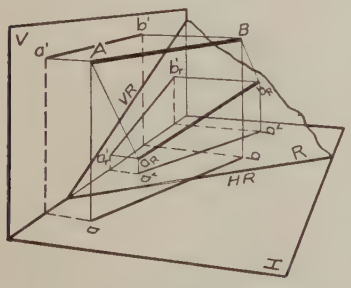


Fig. 185. The Method of Describing a Line in Space and its Representation upon a Plane in Space.

Example. Let it be required to describe the representation of the given line AB upon the given plane R, Fig. 186.

Since AB is a straight line it is necessary to describe only its end points in order to fix its complete graphic description. If the

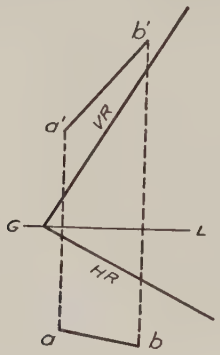


FIG. 186.

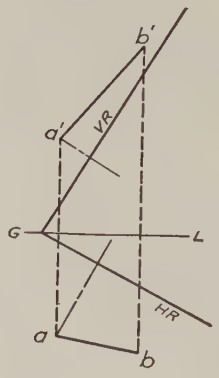


FIG. 187.

line is projected orthographically, the rays which create the representation of these end points may be readily established as they must be perpendicular to the plane. Fig. 187 shows the repre-

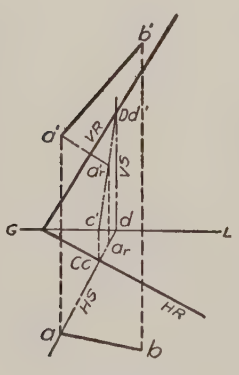


FIG. 188.

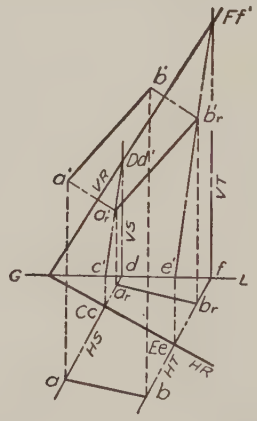


FIG. 189. A Completed Description of the Representation of a Line in Space upon a Plane in Space.

sentations of the projecting ray for the point A. Fig. 188 shows how the description of the intersection of this ray with the given plane is determined and notated. By establishing a similar ray for the point B and determining the description of the point at

which it intersects the given plane, the required description may be completed, see Fig. 189.

If the given line were curved instead of straight it would be necessary to treat several points of the line in order to establish the desired descriptions.

60. Intersections of Planes with Solids — Preliminary Consideration. — The character of the line along which a plane intersects the surface of a solid depends upon the nature of the solid and the arrangement of the plane relative to the solid, see Fig. 190. Like all lines of intersection, this line is composed of points common to the two surfaces and is therefore determined by the general method explained in Art. 52.

The success with which the general scheme of solution is applied to this problem is dependent upon two facts: (1) that the draftsman is at liberty to select the auxiliary surfaces which are employed for the purposes of cutting pairs of intersecting lines from the given surfaces; (2) that it is possible to so choose these auxiliary surfaces that the lines they cut are easily determinable.

Consideration of the general problem of determining the intersections of planes with solids must therefore be postponed until an investigation has been made of so-called *simple lines*. Simple lines are those lines, of the group which can be cut from the surfaces of solids by planes, which are easiest to describe.

61. Simple Lines on Surfaces of Solids. — Two factors are effective in determining the ease with which a line cut from a solid may be determined and described: (1) the arrangement of the cutting plane relative to the solid, because this fixes the nature of the line cut; (2) the position of the solid and of the cutting plane relative to the reference planes, because this fixes the nature of the representations of the line cut.

Consideration of the first factor reveals that only those arrangements of cutting planes and solids which result in straight lines or circles being cut are useful in establishing

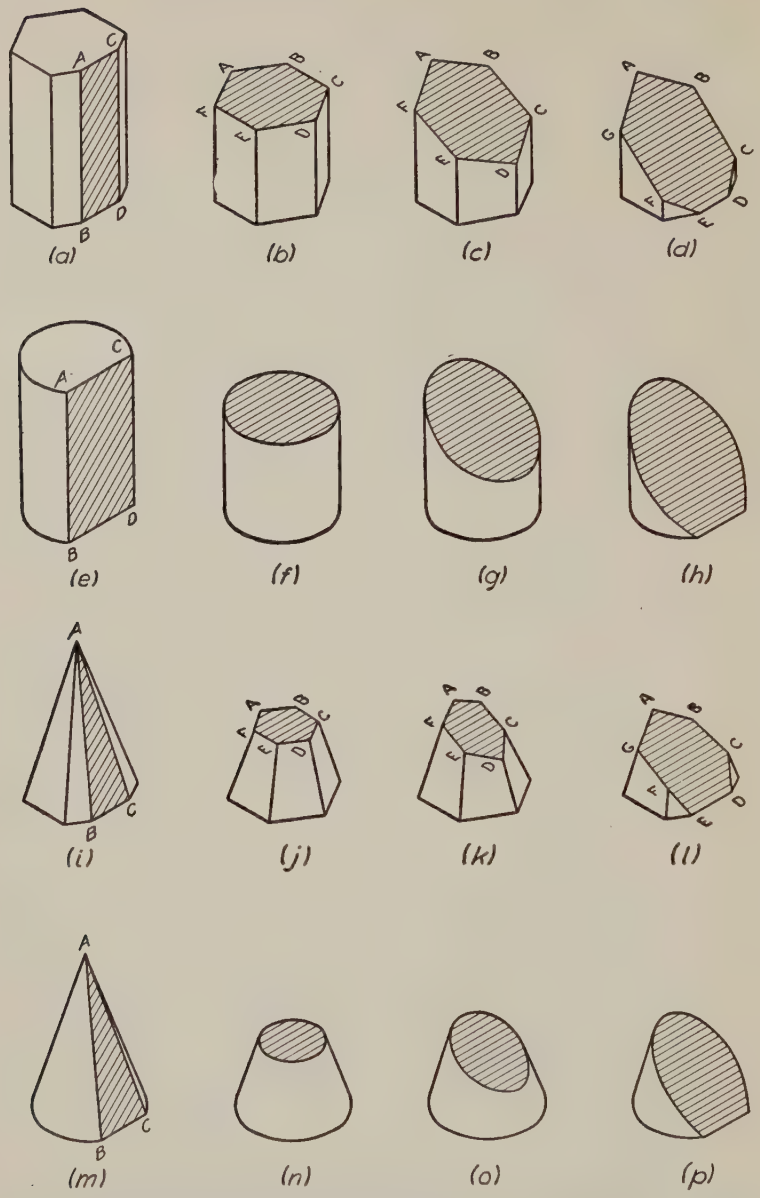


FIG. 190. Types of Lines along which Planes Intersect the Surfaces of Solids.

simple lines because only these lines may be easily described.

Consideration of the second factor reveals the desirability of using auxiliary planes which are perpendicular to a reference plane. This arrangement of an auxiliary plane causes the representation of any line in the plane to coincide with the trace of the plane, see Fig. 191, and thus simplifies the procedure of determining the representations of lines cut by such a plane from other surfaces.

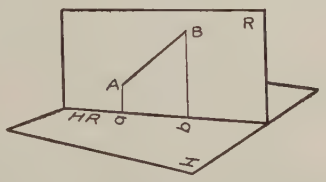


Fig. 191. If a Plane in Space Is Perpendicular to a Reference Plane the Representation of Any Line in the Plane Is Coincident with the Trace of the Plane.

The most desirable arrangement of a solid relative to a pair of reference planes is therefore that which enables straight lines or circles to be cut by auxiliary planes which are perpendicular to one of the reference planes. If straight lines are cut from the surface of the solid, the auxiliary planes may be oblique to the second member of the pair of reference planes without causing difficulty in obtaining representations of the line. If circles are cut, the auxiliary planes must be parallel to the second reference plane as circles may be readily described only when parallel to one of the pair of reference planes.

With this desirable arrangement of solid, reference planes, and auxiliary plane it is not only relatively easy to determine and describe the lines cut from the solid but it is possible to shorten the full procedure of executing the drawings. Short methods of solving those simple problems which are repeatedly applied toward the solution of more complex problems are obviously worthy of attention. However, it is necessary to first investigate the complete procedure of performing such solutions in order to fully appreciate the application of basic theory and to gain that knowledge which is essential to the ability of devising shorter solutions.

62. Full Procedure of Establishing Simple Lines on Surfaces of Solids.

Example 1. Let it be required to cut a simple line from the lateral surface of the prism shown in Fig. 192.

The first step in planning the solution of this problem is to select a cutting plane which will be so positioned relative to the solid and to the reference planes that it will cut simple lines. The plane shown in Fig. 193 meets the requirements established

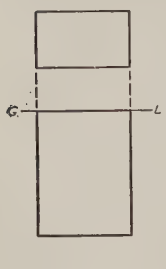


Fig. 192.

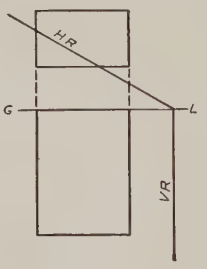


Fig. 193.

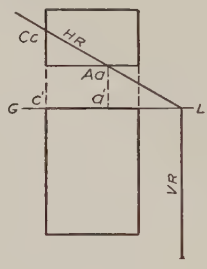


Fig. 194.

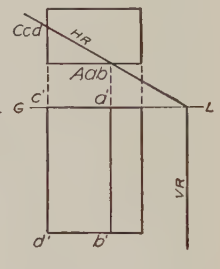


Fig. 195. The Completed Description of the Simple Lines Cut from the Surface of a Solid by a Plane.

in the above discussion as it is perpendicular to a reference plane and cuts straight lines from the surface of the solid.

The second step of the solution is to devise a method of determining the lines cut from the solid by the plane. Because the lines are known to be straight, each may be found by establishing either two of its points or one point and its direction. Upon examination of the data it may be seen that the lines are parallel to the lateral edges of the prism; hence but one point on a line need be found. Since a point is created by the intersection of two lines, it is now necessary to establish a pair of lines which will intersect to create a point on a desired line. One member of such a pair of intersecting lines must be in the surface of the plane and the other in the surface of the solid. Inspection of the data, Fig. 193, shows that such a pair of lines are already established. The H-reference plane contains a line of the plane (its H-trace) and also contains a base of the solid. The H-trace of the plane intersects the base

lines of the solid at two different points, see Fig. 194, one on each of the two lines cut from the lateral surface of the prism. Since the direction of the lines is known, their representations may be added to the drawing, see Fig. 195.

Example 2. In this example a prism similar to the one used in the preceding problem is positioned so that neither base is contained by a reference plane, see Fig. 196. This figure also illustrates the plane selected for the purpose of cutting the simple lines from the lateral surface of the prism.

Consideration of this problem results in recognition of the fact that it is essentially the same as the preceding problem and that it may be solved by the same general method. It differs from

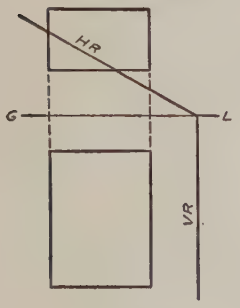


Fig. 196.

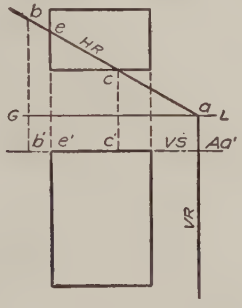


Fig. 197.

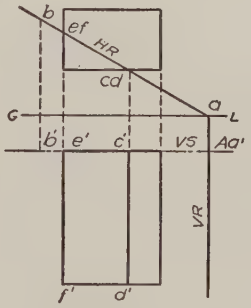


Fig. 198. The Completed Description of the Simple Lines Cut by a Plane from the Surface of a Solid when a Face of the Solid Does not Lie in a Primary Reference Plane.

the first, however, in that a pair of intersecting lines are not already available for determining a point on the line, because a base of the prism does not lie in a reference plane. Evidently a new plane must be assumed for the purpose of establishing the necessary intersecting lines. If this new plane is taken parallel to H and through one base of the prism, it may be used in exactly the same manner as the H-plane was used in Example 1. The new plane cuts a line from the original plane and contains a base of the prism. Two points are created by the intersection of the base lines with the line cut from the original plane, see Fig. 197. These points together with the known direction enable the draftsman to com-

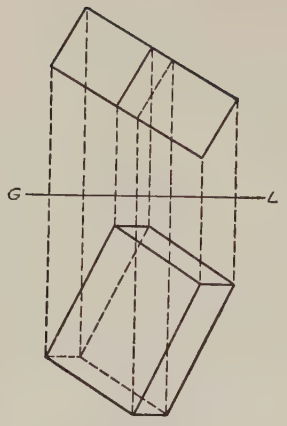


FIG. 199.

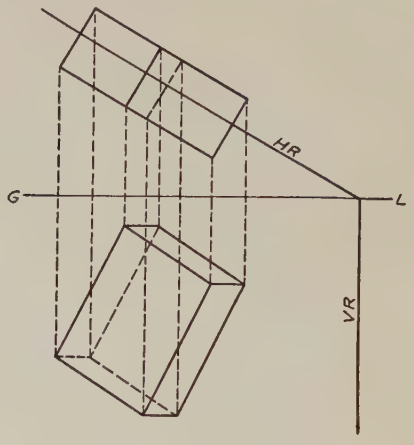


FIG. 200.

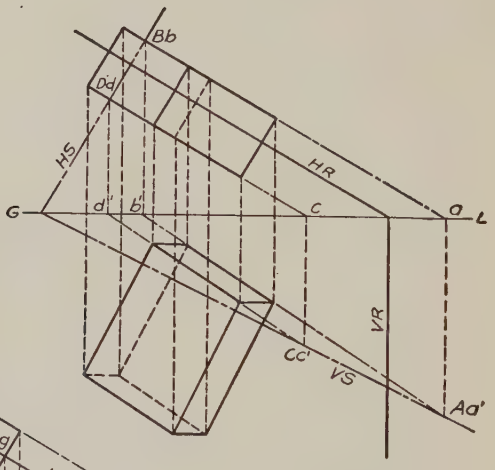


FIG. 201.

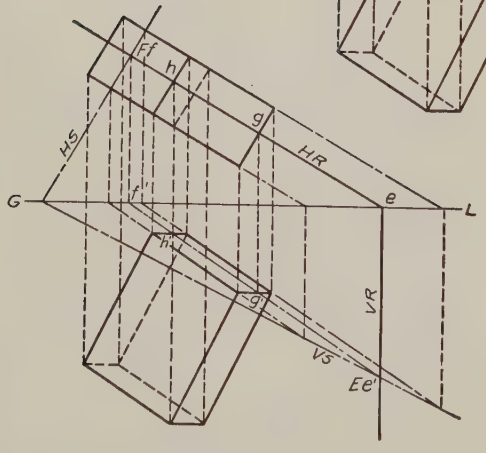


FIG. 202.

plete the description of the two simple lines which are cut from the lateral surface of the prism, see Fig. 198.

Example 3. Let it be required to cut simple lines from the lateral surface of a prism which is positioned as shown in Fig. 199.

The requirements of a plane to cut simple lines are obviously met by the plane shown in Fig. 200.

Examination of the problem shows it to be identical in nature with Example 2 and that the same method of solution may be applied. The task of establishing a new plane through a base of the prism is somewhat more involved than in the previous prob-

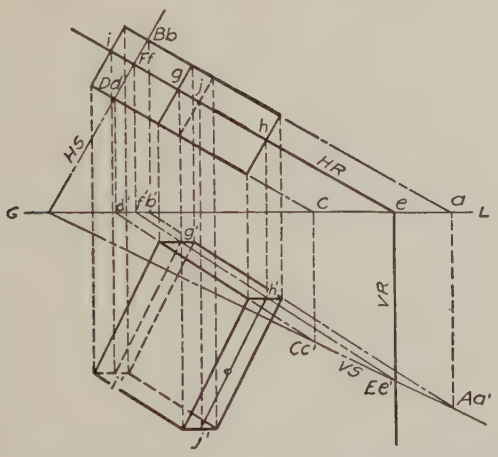


FIG. 203. The Completed Description of the Simple Lines Cut by a Plane from the Surface of a Non-Orthometrically Positioned Solid.

lem; however, this may be accomplished by passing the plane through two parallel edge lines of the base, see Fig. 201. Once the new plane is established it is a simple matter to determine the line it cuts from the original plane and to find the points at which this line intersects the base lines, see Fig. 202. The fact that the directions of the simple lines are parallel to the lateral edges of the prism enables the draftsman to complete their description as shown in Fig. 203.

Example 4. In Examples 1 and 2 the given prism was *orthometrically positioned* relative to both primary reference planes, that is, two of the prism's three principal dimensions were parallel to each reference plane. In Example 3 the prism was *non-*

orthometrically positioned relative to the primary planes, as no two of its principal dimensions were parallel to either reference plane.

A comparison of the drawings resulting from the full procedure of solving these problems shows that it is advantageous to have the solid orthometrically positioned, see Figs. 198 and 203. The object of this illustrative example is to demonstrate the possibility of reducing a problem which involves a non-orthometrically positioned solid to the simpler type of problem which involves an orthometrically positioned solid. This is accomplished by establishing a new pair of reference planes in such a manner as to satisfy the requirements of orthometric positioning.

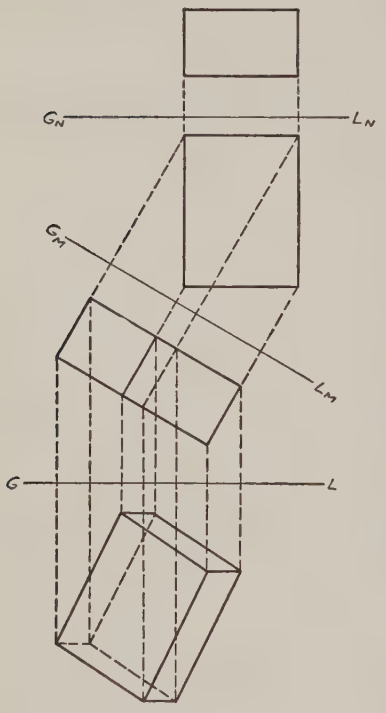


Fig. 204.

Let it again be required to cut simple lines from the lateral surface of the prism as given in Fig. 199. The first step of the solution consists of establishing a new pair of reference planes, *M* and *N* in Fig. 204, which satisfy the requirements of orthometric positioning relative to the prism. The problem may now be solved by employing the new pair of reference planes, see Fig. 205, and the drawing may be completed by adding the description of the cutting plane and the lines to the primary reference planes, see Fig. 206. The task of establishing the primary traces of the cutting planes is most easily performed by first establishing the primary representations of the simple lines; since these lines determine the plane, its traces may be readily established.

63. Shorter Procedure of Establishing Simple Lines. — Referring to Figs. 195, 198 and 203, it may be seen that in

every case one representation of the simple line is coincident with a trace of the auxiliary plane that cut it. This coincidence occurs on that reference plane to which the cut-

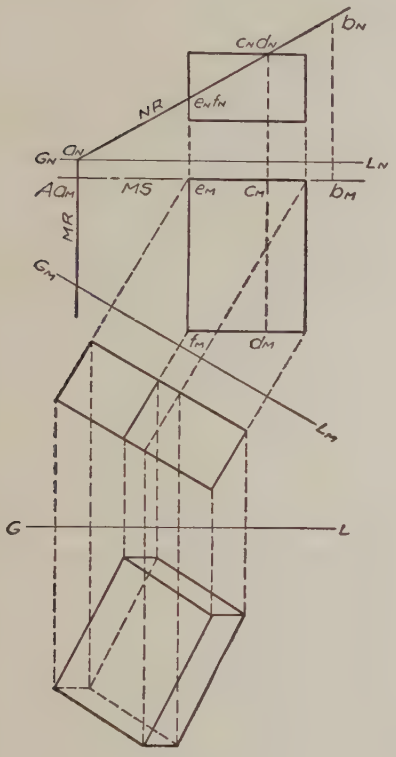


Fig. 205.

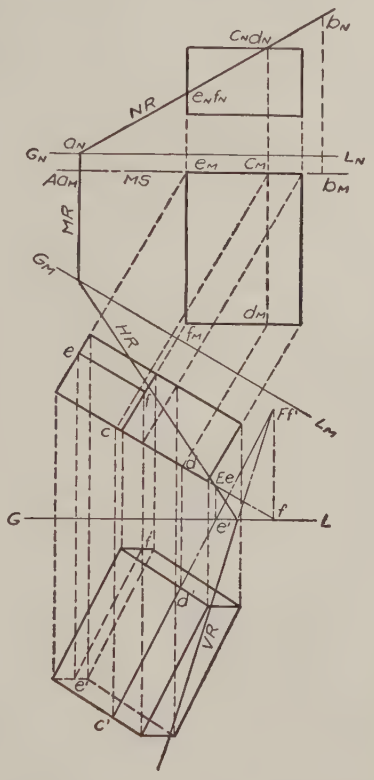


Fig. 206. A Completed Description of the Simple Lines Cut by a Plane from the Surface of a Non-Orthometrically Positioned Solid. An Additional Pair of Reference Planes, Orthometrically Positioned Relative to the Solid Are Used in this Description.

ting plane is perpendicular. Obviously these representations may be located on a drawing immediately after the cutting plane has been established, see Fig. 207. Representations of these lines on other reference planes may then

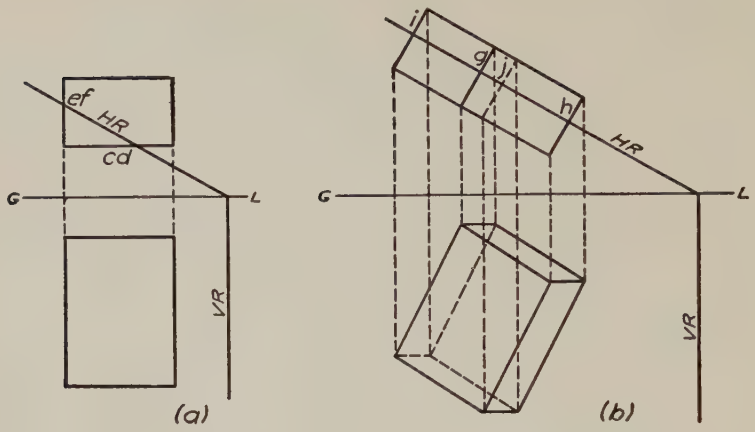


Fig. 207.

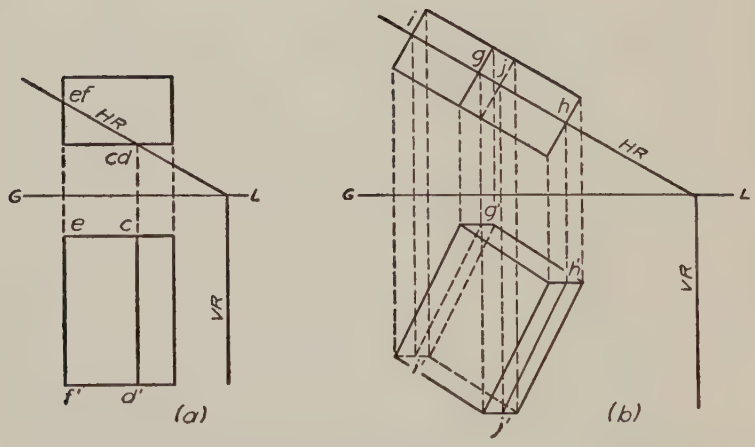


Fig. 208. Completed Descriptions of Simple Lines Cut from the Surfaces of Solids. The Shorter Method of Describing the Lines Is Used in these Examples.

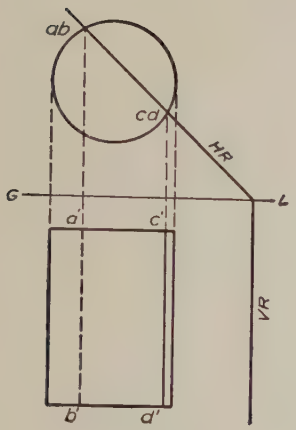


FIG. 209.

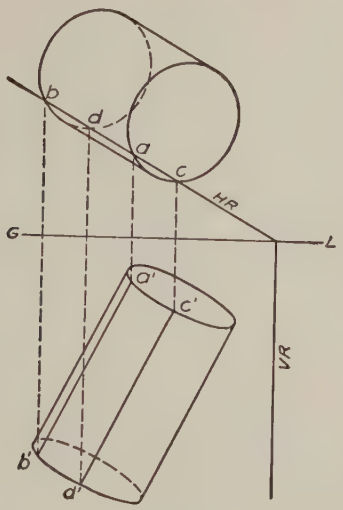


FIG. 210.

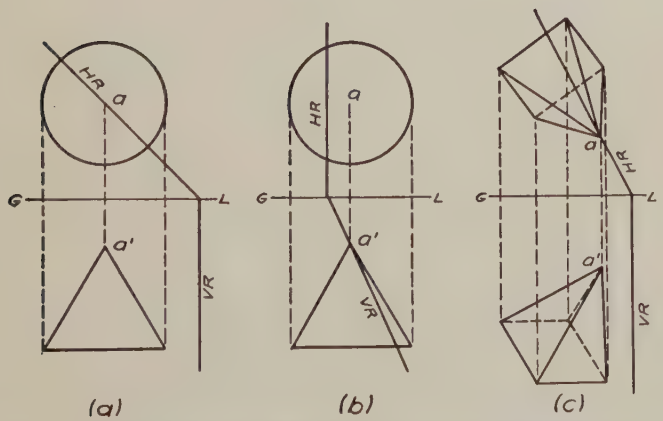


FIG. 211. Positioning Cutting Planes to Cut Simple Straight Lines from the Surfaces of Solids.

be readily established because their end points are known to lie on base lines of their respective solids, see Fig. 208.

A comparison of Fig. 208*b* with Fig. 203 clearly indicates the advantages of this shorter method of locating the representations of simple lines.

Figs. 209 and 210 illustrate respectively the application of this shorter method to the determination of simple lines cut from orthometrically and non-orthometrically positioned cylinders.

Fig. 211 shows several arrangements of reference planes, solids and cutting planes which result in simple lines being cut from the surfaces of the solids. Fig. 212 illustrates how

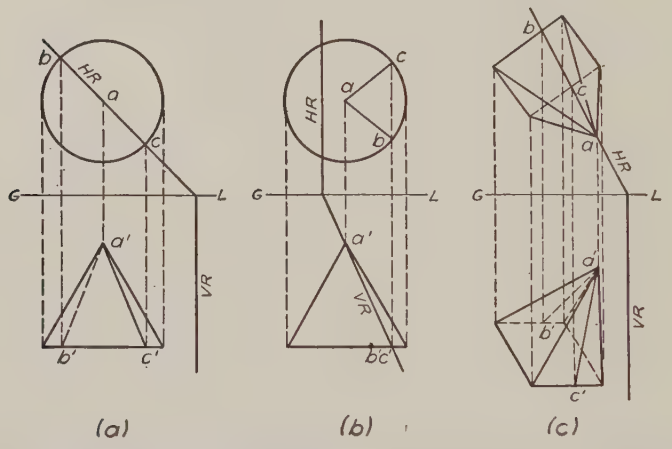


Fig. 212. Descriptions of Simple Straight Lines Cut from the Surfaces of Solids by Planes.

these simple lines are determined and described by the shorter method. Note that a cutting plane must contain the apex of a cone or pyramid in order to cut straight lines from their surfaces.

Circles must be parallel to a reference plane in order to be considered simple in nature. Hence simple circular lines can be cut from a circular cone only when the cone is orthometrically positioned, see Fig. 213*a*. Fig. 213*b* illus-

trates a simple line cut from a pyramid in a similar manner. Since but relatively few points need be determined to establish this line it may be considered as simple even when the pyramid is non-orthometrically positioned.

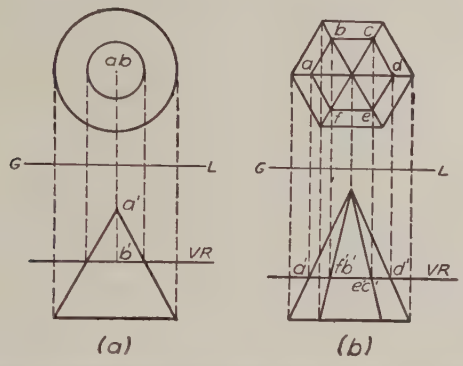


FIG. 213. Simple Intersections of Planes with Solids. These Intersections May Be Classed as Simple Lines.

If a cutting plane is perpendicular to a reference plane it may be oblique to the axes of prisms and pyramids and still cut simple lines from their surfaces, see Fig. 214*a* and *b*.

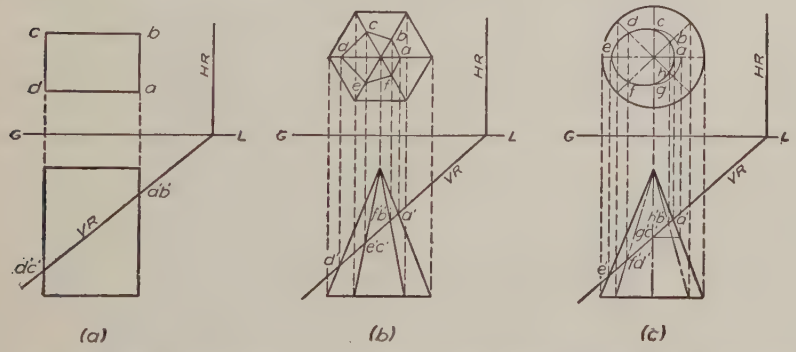


FIG. 214. Simple Intersections of Planes with Solids. The Line Cut from the Cone at (*c*) Cannot Be Classed as a Simple Line.

However, if such a cutting plane is oblique to the axis of a cone or cylinder the line cut can not be considered simple in nature, see Fig. 214*c*.

64. Intersections of Planes with Solids — The General Problem. — The general problem of determining the intersection of a plane with a solid is solved by the basic method explained in Art. 52. In applying this basic method auxiliary planes are employed to cut *simple lines* from the surface of the solid and desirable lines from the given plane. Pairs of these lines intersect to create points on the line of intersection.

The ability to select auxiliary planes which will cut simple lines from given solids and the ability to determine the descriptions of the lines cut by these planes is essential to planning and executing solutions of these problems.

The first step in planning the solution of this general problem consists of examining the given data and deciding upon the manner of passing the auxiliary planes. For example, let it be required to describe the intersection of the cylinder and plane shown in Fig. 215. It should be first noted upon examination of the drawing that an auxiliary plane perpendicular to H would cut simple lines from the lateral surface of the cylinder. It should then be seen that if the auxiliary plane were taken parallel to V it would not

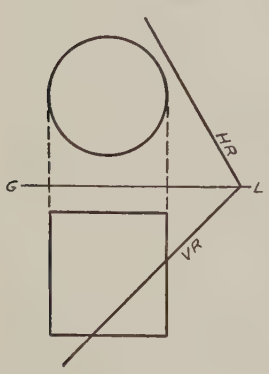


Fig. 215.

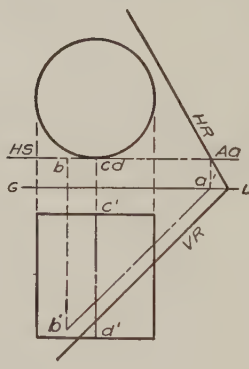


Fig. 216.

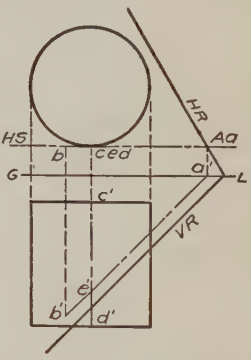


Fig. 217. The Completed Description of One Point on the Intersection of an Oblique Plane with a Solid.

only cut simple lines from the surface of the cylinder but it would also cut a line from the given plane which could be determined by the point and direction method, see Fig. 216. Since the intersection of the line cut from the cylinder with the line cut from the given plane is one point on the desired line of intersection, the representations of this point would be notated on the drawing, see Fig. 217. If parallel auxiliary planes of this nature are employed when determining additional points on the line of intersection the descriptions of the lines cut from the given plane could be found with less effort and the resulting drawing would be more orderly than would be possible with any other arrangement.

The scheme of a solution is always devised in the order demonstrated by the above example. However, when applying a scheme to the actual execution of a drawing it is

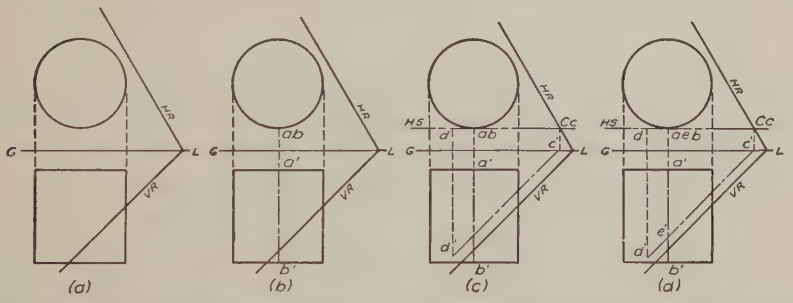


FIG. 218. An Order of Execution which Permits the Use of Preferred Simple Lines for Creating Points on the Intersection of a Plane with a Solid.

often found desirable to reverse this order. For example, instead of first establishing the auxiliary planes it is often better to establish a preferred selection of simple lines on the surface of the solid which should be cut in preference to others, and then to establish the auxiliary planes so as to cut these particular lines as well as desirable lines from the given plane. Of course the selection of the preferred simple lines must be made in accordance with a previously planned scheme of solution. Fig. 218 illustrates the application of this procedure to the execution of a drawing on which but

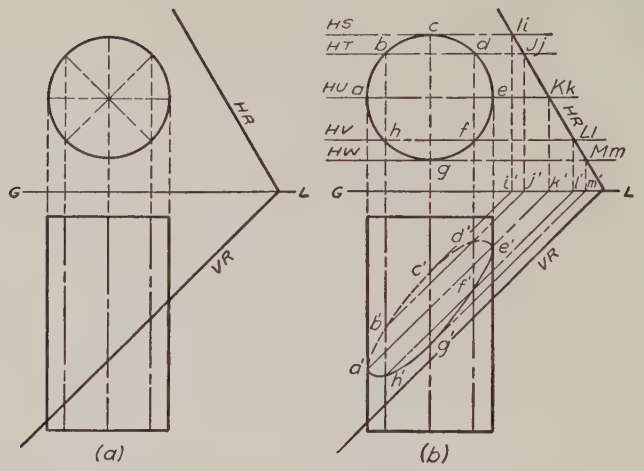


FIG. 219. A Description of the Intersection of a Plane with a Cylinder. Preferred Simple Lines Are Used in this Description.

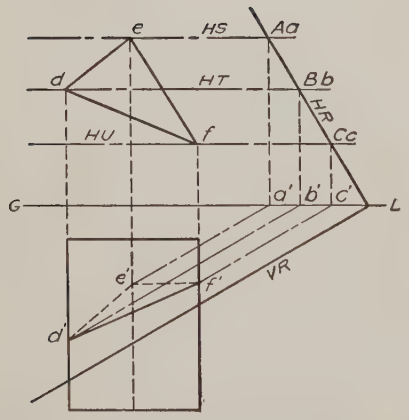


FIG. 220. A Description of the Intersection of a Plane with a Prism in which Preferred Simple Lines Are Employed.

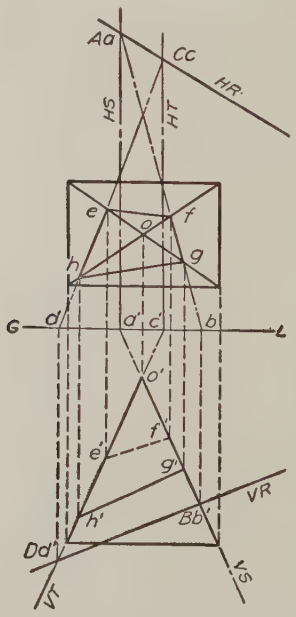


FIG. 221. A Method of Employing Preferred Simple Lines when Describing the Intersection of a Plane with a Pyramid.

one point of the line of intersection is determined. Fig. 219 illustrates the orderly appearance of a completed drawing executed in accordance with this procedure.

Since a cylinder may be considered to be a prism having an infinite number of lateral faces, problems involving prisms are solved by the same scheme as employed for similar cylinder problems. However, the execution of a prism problem differs from one of a similar cylinder problem in that the draftsman is not offered a choice when establishing the preferred simple lines. Referring to Fig. 220, it may be seen that the lateral edges of the prism intersect the given plane at critical points on the line of intersection and that, since these points are sufficient to determine the entire line of intersection, only the edge lines should be chosen when establishing the preferred simple lines.

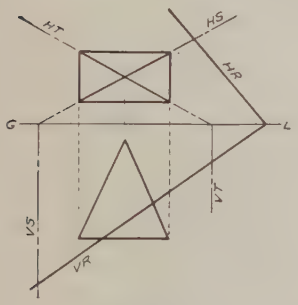


Fig. 222.

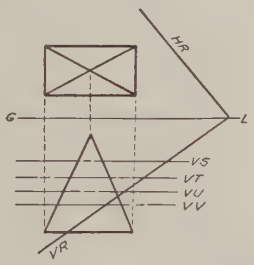


Fig. 223.

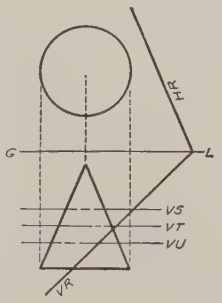


Fig. 224.

The desirable parallel arrangement of auxiliary planes which is possible when dealing with cylinders and prisms cannot be realized when dealing with pyramids and cones, because lines on the lateral surfaces of these solids are not parallel. Fig. 221 illustrates a drawing made to describe the intersection of a pyramid with an oblique plane. In this solution the lateral edges are recognized as preferred simple lines and auxiliary planes perpendicular to V were passed through them. Fig. 222 suggests a second method of passing the auxiliary planes through the preferred simple lines.

An arrangement of auxiliary planes which do not cut

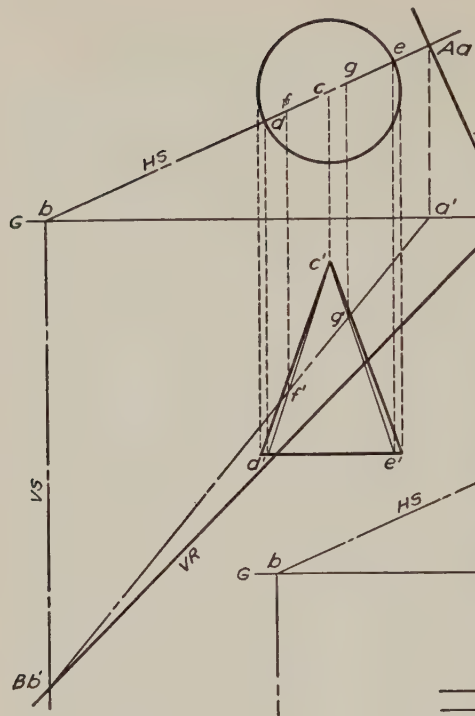


FIG. 225.

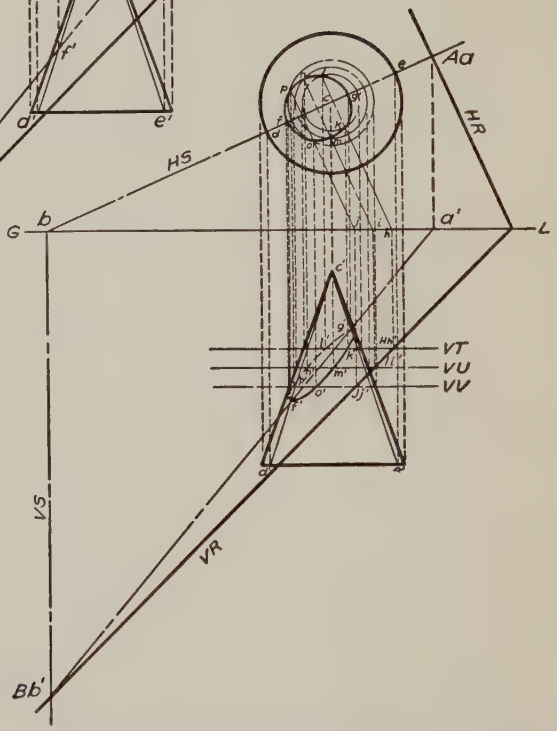


FIG. 226. A Completed Description of the Intersection of an Oblique Plane with a Cone.

preferred simple lines from a pyramid is shown in Fig. 223. This arrangement is far inferior to those illustrated in Figs. 221 and 222; however, since this method of passing auxiliary planes would result in simple circles being cut from an orthometrically positioned circular cone, see Fig. 224, it may be used for the solution of cone problems. When executing a solution of a cone problem by this scheme it is

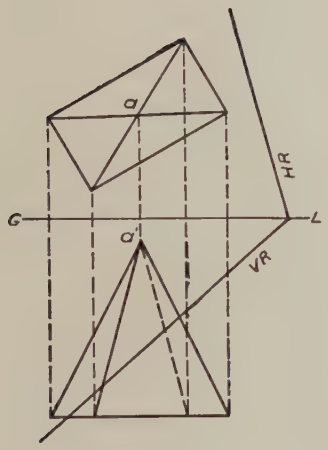


Fig. 227.

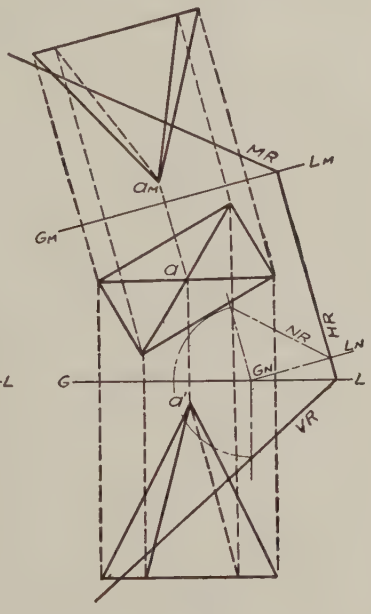


Fig. 228.

desirable to determine first the extreme high and low points on the line of intersection, for the placing of the auxiliary planes may then be planned to cut preferred lines from the cone. These high and low points may be found by using one auxiliary plane of a different order, that is, a plane through the axis of the cone and perpendicular to the given plane. This auxiliary plane cuts elements from the cone which intersect the given plane at the required points, see Fig. 225. Auxiliary planes of the original order may then be used to determine the intermediate points, see Fig. 226.

The problem of determining the intersection of an oblique plane with a solid may be reduced to the relatively simple problem of determining the intersection of the solid with a plane which is perpendicular to a reference plane by establishing a new pair of reference planes so as to bring about this desirable condition. Let Fig. 227 be the given data for such a general problem. Fig. 228 illustrates a method of establishing a new reference plane perpendicular to the given plane. This new reference plane and the H-plane form a pair of reference planes which satisfy the requirements of the simpler solution. Representations of points along the intersection of the plane with the solid are first located on the new reference plane and then added on the primary planes, see Fig. 229.

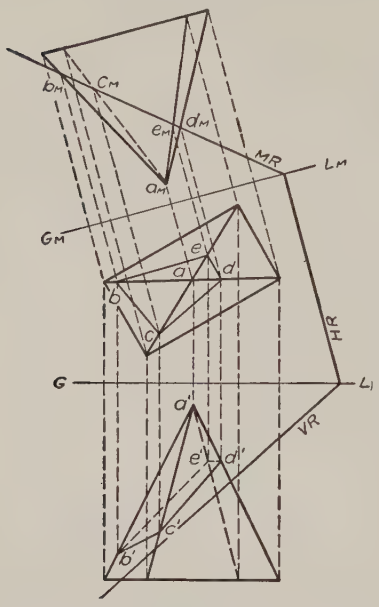


FIG. 229. A Description of the Intersection of an Oblique Plane with a Pyramid. This Problem in Description is Solved by Reducing it to a Simpler form.

Since the base of a solid is a plane surface only a straight line can be cut from it by a plane which intersects the solid, see Figs. 190*d*, *h*, *l* and *p*. To determine the line along which a plane cuts the base of a solid it is first necessary to establish the plane of the base, see Fig. 230*a*. The intersection of this base plane with the given plane is then found, see Fig. 230*b*. The portion of this intersection which lies within the limits of the base is the line cut from the base by the given plane.

65. Intersections of Solids — Preliminary Consideration. — The component points of a line along which the surfaces of two solids intersect are determined in the same

manner as points on the intersection of a plane with a solid, each point being created by the intersection of a pair of

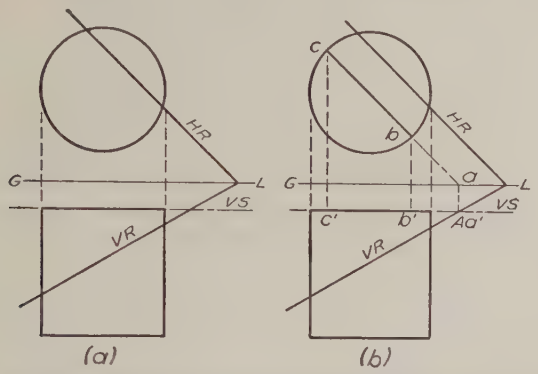


Fig. 230. A Description of the Line Cut from the Base of a Solid by an Oblique Plane.

simple lines which are cut, one from the surface of each solid, by an auxiliary third surface.

The following example illustrates the general method of determining a point on the line of intersection of two solids.

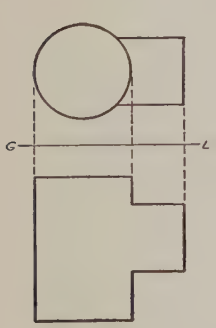


Fig. 231.

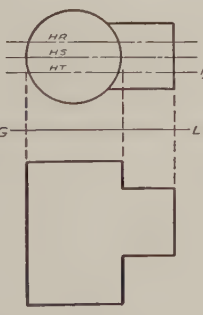


Fig. 232.

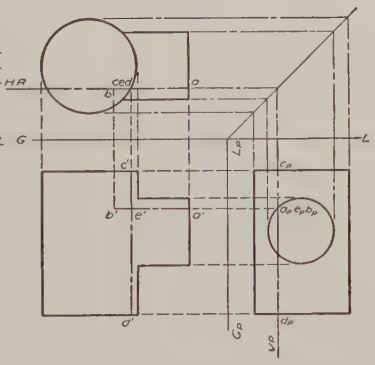


Fig. 233. A Completed Description of One Point on the Line along which Two Solids Intersect.

Let the combination of solids shown in Fig. 231 be the given data. To cut a pair of simple lines, one from the surface of each of the solids, the auxiliary planes must be parallel

to V, see Fig. 232. The intersection of these simple lines is a point on the line of intersection of the solids, see Fig. 233.

66. Establishing Planes to Cut Preferred Simple Lines from Solids. — The first requirement to be met when devising a scheme for solving a solid intersection problem is that the lines cut from the surfaces of the solids by the auxiliary surfaces must be easy to determine and to describe. It has been previously explained in the discussion of intersections of planes with solids that cutting planes which are perpen-

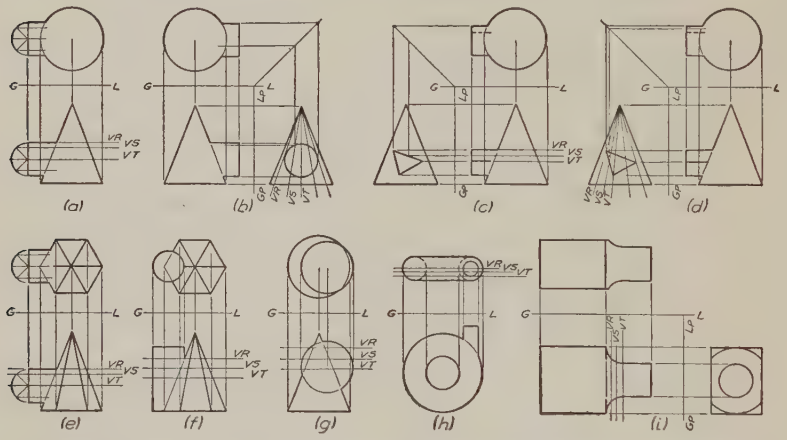


Fig. 234. Examples of Combinations of Solids from which Simple Lines May Be Cut by Planes which Are Perpendicular to a Reference Plane.

dicular to a reference plane offer a considerable advantage in this respect. It has also been shown that straight lines and circles should be cut from the intersecting surfaces, rather than lines of more complex natures, and that when circles are cut their planes should be parallel to a reference plane.

The possibility of cutting either straight lines or desirable circles from the surfaces of two intersecting solids by planes taken perpendicular to a reference plane, depends upon the nature of the solids, their arrangement relative to each other and upon the arrangement of the combination of solids relative to the reference planes.

A number of combinations of solids which are so placed relative to the primary reference planes that desirable lines can be cut from their surfaces by auxiliary planes which are perpendicular to a reference plane, are illustrated in Fig. 234.

If, due to the arrangement of a combination of solids relative to the primary reference planes, it is impossible to cut desirable lines from their surfaces by auxiliary planes which are perpendicular to one of the primary planes, the draftsman must choose between two alternative procedures. One procedure consists of establishing a new pair of reference planes for working purposes, so positioned relative to the combination of solids that they may be used with auxiliary planes of the desired perpendicular arrangement. In the second procedure the primary reference planes are used for working purposes and auxiliary planes are established so as to cut desirable lines from the surfaces of the solids regardless of the position of these planes relative to the primary reference planes. The following example illustrates the application of these two procedures to the solution of a problem.

Let it be required to describe the intersection of the solids shown in Fig. 235. A preliminary examination of the given data shows that it is impossible to cut pairs of simple lines from the surfaces of the two solids by auxiliary planes which are perpendicular to either of the primary reference planes. The next step in planning the solution is to establish mentally a set of auxiliary planes which would cut simple lines. This is usually accomplished by first selecting the types of lines which are desirable and then testing the possibility of cutting them by planes. Since straight lines are in general to be preferred to others, it is usual practice to test first the possibility of setting up planes which would cut this type of line from each of the surfaces of the solids. Examination of the given data shows this to be possible, because planes may be established so as to pass through the apex of the cone and be parallel to the lateral edges of the prism. Auxiliary planes of this nature, excepting one, would be

oblique to V and H; hence the draftsman must choose between the two procedures discussed above.

If it is decided to establish a new pair of reference planes, one of the pair should be perpendicular to the lateral edges

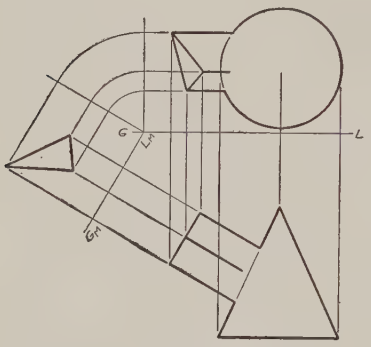


Fig. 235.

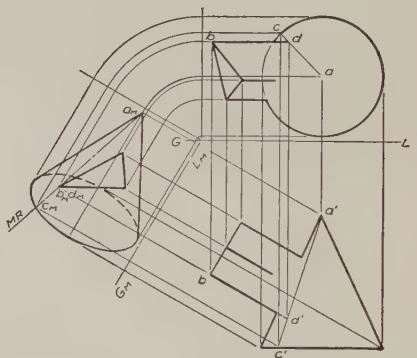


Fig. 236. A Description of One Point on the Line of Intersection of Two Solids. The Use of a Cutting Plane which is Perpendicular to a Reference Plane Is Made Possible in this Description by Establishing a New Pair of Reference Planes.

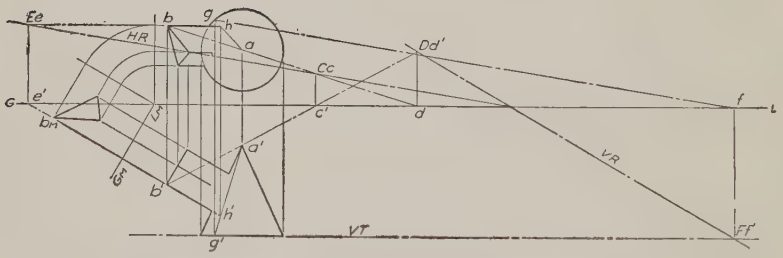


Fig. 237. A Description of One Point on the Intersection of Two Solids. The Cutting Plane Used in this Description Is Taken Oblique to the Reference Planes.

of the prism because this would bring about the desirable perpendicular relation of the auxiliary planes and a reference plane. Fig. 236 illustrates such a new pair of reference planes (V and M) and also shows how they may be employed with an auxiliary plane for determining one point on the desired line of intersection.

If it is decided to follow the second procedure, that of using the primary planes as working planes, each auxiliary plane must be established by passing it through a pair of *intersecting* lines, one of which is a line of the prism and the other a line passing through the apex of the cone, see Fig. 237. The method of determining the simple line cut from the cone by the auxiliary plane is also illustrated by this drawing. The intersection of the two lines cut from the surfaces of the solids creates one point on the desired line of intersection.

67. Selection of Preferred Points for Determining Lines of Intersection. — Since a draftsman seeks to establish the *representations* of a line of intersection rather than the line itself, he bases his choice of the points to be used in determining the line upon the advantages they offer in fixing the representations of the line rather than their ability to fix the actual line in space. The number of points employed in determining a line of intersection should be the minimum consistent with accuracy because it becomes increasingly difficult to execute and read drawings as they grow congested with delineation.

Evidently the planning of a solution of an intersection problem is not completed by merely deciding upon a method of establishing the auxiliary planes so as to cut desirable types of lines from the surfaces of the solids. It is also necessary to plan the number and locations of the points to be used in determining the line, so that the line will be accurately and clearly represented on the resulting drawing.

The problem of determining the intersection of two prisms, see Fig. 238, offers a simple example of the need of carefully selecting the points employed. A preliminary examination of the given data shows that auxiliary planes parallel to V should be used as they cut lines from the prisms which are most easily determined and described. Further examination of the problem reveals that the actual line of intersection is composed of several straight lines and that it is essential to determine only those critical points on the

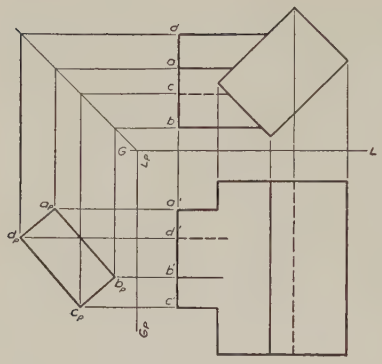


FIG. 238.

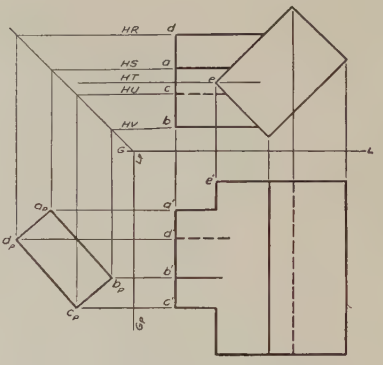


FIG. 239. Selecting the Preferred Simple Lines which May Be Used for Determining Critical Points on the Intersection of Two Solids.

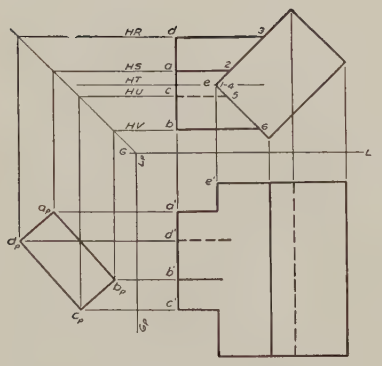


FIG. 240. Notating the Available Representations of Critical Points.

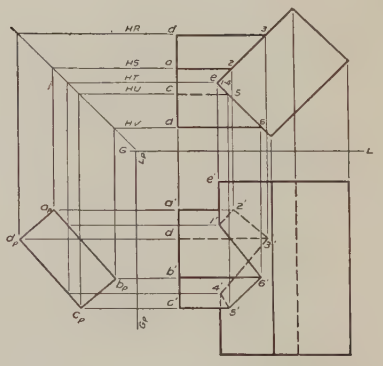


FIG. 241. The Completed Description of the Intersection of Two Solids of such Nature that only Critical Points Need Be Determined to Describe the Intersection.

line of intersection at which these component lines intersect. Again inspecting the given data it may be seen that there are six of these critical points, one being created by each of the four lateral edges of the horizontal prism as they intersect the surface of the vertical prism and the remaining two by an edge of the vertical prism as it enters and leaves the surfaces of the horizontal prism. As a result of thus planning the scheme to be used in solving the problem the draftsman is prepared to undertake intelligently the task of executing the drawing. The proper number of auxiliary planes may first be established, see Fig. 239. Next, to simplify execution, notation should be assigned the H-representations of the critical points in accordance with the order of encountering them when tracing the line of intersection, see Fig. 240. The V-representations of the lines cut by the auxiliary planes are then determined and the V-representations of the critical points are also found. The sequence of the notation of these points assists the draftsman to add the representations of the component lines which form the line of intersection, see Fig. 241.

The given data for a second example in the selection of preferred points for determining a line of intersection is given in Fig. 242. An examination of these solids shows that auxiliary planes, parallel to V may again be used for cutting the simple lines. The points at which the lateral edges of the prism intersect the surface of the cylinder are obviously critical points on the line of intersection and should be determined, see Fig. 243. Other points must be established as illustrated in Fig. 244, but the draftsman should first investigate the general nature of the representations of the line of intersection so as to select the minimum number of desirably positioned points. It may be readily observed that the actual line of intersection is made up of three curved lines having for their end points the three critical points created by the lateral edges of the prism striking the surface of the cylinder. The technique of selecting points for determining the representation of a curved line is illustrated

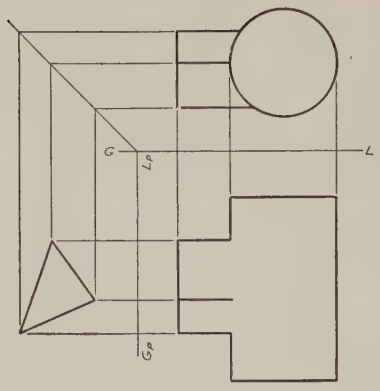


FIG. 242.

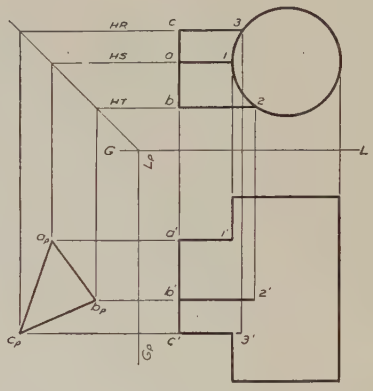


FIG. 243. Describing the Critical Points on the Intersection of Two Solids.

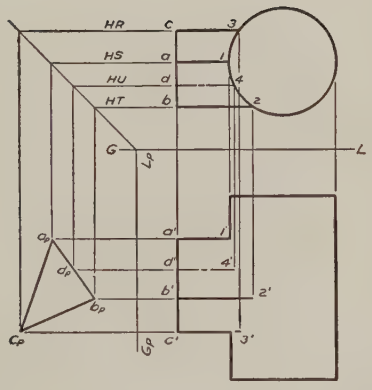


FIG. 244. The Method of Describing Points in Addition to the Critical Points of the Intersection of Two Solids.

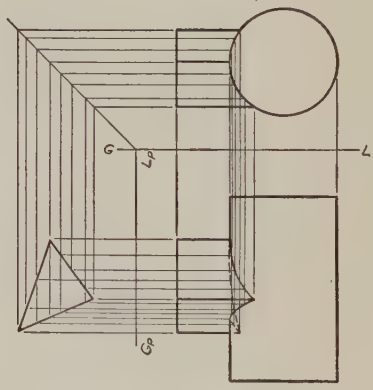


FIG. 245. A Completed Description of the Intersection of Two Solids which Illustrates a Proper Selection of Preferred Points for Determining the Representations of the Intersection.

in Fig. 246. The selection of points shown at *a* is such that their representations definitely fix the representation of the line and at the same time unnecessary work and congestion of the drawing is avoided. The selection of points shown at *b* fix the representation of the line but require unnecessary work to be performed and may cause confusion of delineation. The selection shown at *c* although equal in number to that shown at *a* are poorly chosen and do not definitely fix the representation of the line. Fig. 245 shows a completed drawing for the problem and illustrates good technique in point selection.

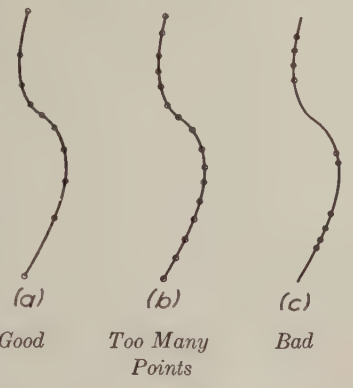


Fig. 246. Preferred Points on a Line Are Selected for their Ability to Fix the Representations of the Line.

68. Use of Spheres as Auxiliary Surfaces. — A point on the intersection of two solids is always created by the intersection of two lines which are cut, one from the surface of each solid, by a third or auxiliary surface. Planes, being the simplest of surfaces, enjoy the widest usage as these auxiliary surfaces. However, the sphere also lends itself advantageously to this purpose under those conditions which cause simple circles to be formed by its intersection with the surfaces of the given solids.

An investigation shows that circles are cut from the surface of a sphere by plane surfaces, regardless of their arrangement relative to the sphere. It is also found that if the center of the sphere is on the axis of a surface of revolution the intersection of the sphere with that surface is a circle, see Fig. 247.

Evidently if the axes of two solids of revolution intersect, a sphere with its center at the intersection of the axes would cut a circle from the surfaces of each of the solids. If the solids are so positioned, or may be so positioned, relative

to a pair of reference planes that these circles may be readily described, concentric spheres of this nature could be used as auxiliary surfaces when determining the intersection of the solids. Fig. 248 illustrates how one such sphere may be

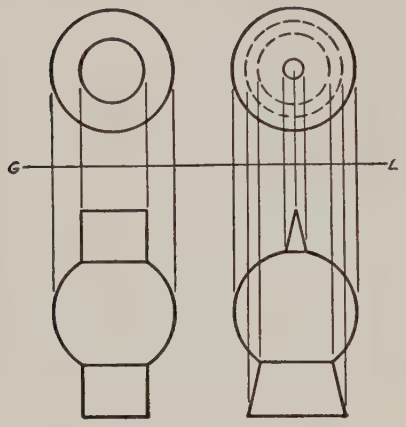


Fig. 247. Spheres May Be Used for Cutting Simple Circles from Surfaces of Revolution.

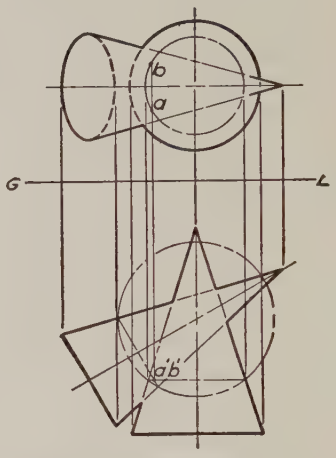


Fig. 248. A Description of Two Points on the Intersection of Two Surfaces of Revolution. A Sphere Is Used to Cut the Lines which Intersect to Create these Points.

used for determining a point on the intersection of two circular cones whose axes intersect.

69. Additional Notes on Drafting Technique. — A change in delineation policy was introduced in Fig. 234, narrow solid lines being used for ruled projectors and construction work instead of lines of broken structures. This policy will be followed except in those cases where confusion might result.

Practical short methods of establishing points on the bases of circular cylinders through which to draw preferred simple lines are also introduced by Fig. 234. These short methods are simply contractions of the complete method of establishing the descriptions of such points.

Fig. 249 illustrates the complete methods of describing

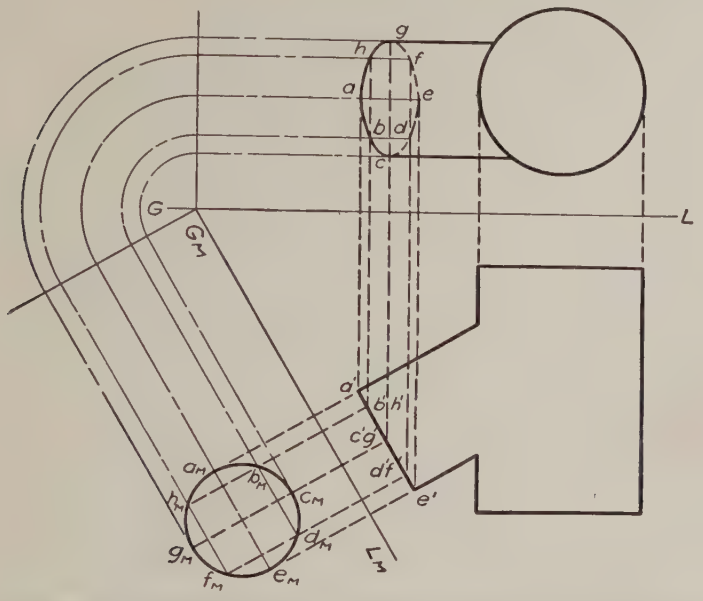


FIG. 249. The Full Procedure of Describing Points on the Base of an Oblique Cylinder.

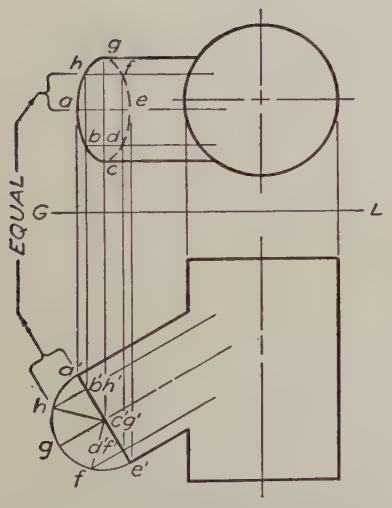


FIG. 250. The Shorter Method of Describing Points on the Base of an Oblique Cylinder.

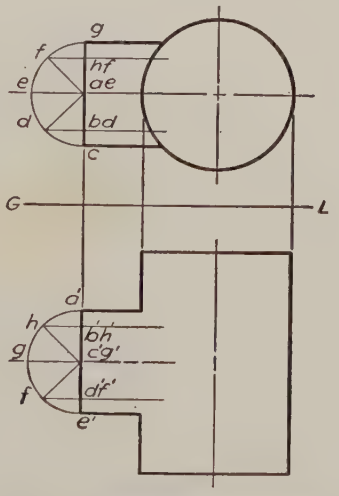


FIG. 251. The Shorter Method of Describing Points on the Base of a Horizontal Cylinder.

points on the base of an oblique cylinder. The contraction of this procedure to the shorter shown in Fig. 250 is obvious.

A short method of establishing points on the base of a horizontal cylinder is shown in Fig. 251. This method is not applicable unless the points are arranged symmetrically with respect to the vertical and horizontal diameters of the cylinder's base, as the two contracted additional views would not be alike unless this symmetrical arrangement existed. Because symmetrical arrangements may be established by dividing the circumferences of the base with the standard 45° and 30–60° triangles, this scheme enjoys wide usage.

CHAPTER IX

DESCRIPTION OF TANGENT RELATIONS

70. Introduction. — The general problem of describing tangent relations of lines and surfaces is attacked, as are all problems, by first compiling from the available information all facts pertinent to the solution. Pertinent facts in this case comprise those which enable the conditions of tangency to be defined in terms of simple in-space relations of points, lines and planes; relations with which the draftsman is already familiar and is capable of describing.

71. Definitions. — A *straight line* is *tangent to a plane curve* when in that limiting position of a secant at which the points of secancy are coincident, see Fig. 252.

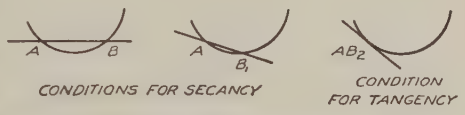


Fig. 252. A Straight Line Tangent to a Curve.

A *straight line* is *tangent to a curved surface* when tangent to a line of that surface, see Fig. 253.

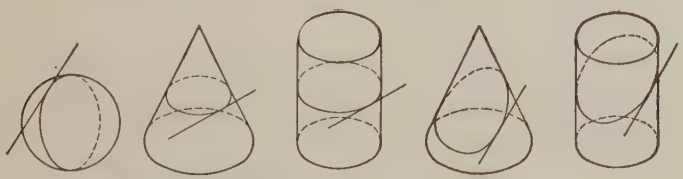


Fig. 253. Straight Lines Tangent to Surfaces.

A *plane* is *tangent to a plane curve* when it is perpendicular to the plane of the curve and contains a straight line tangent to the curve, see Fig. 254.

A *plane* is *tangent to a singly curved surface* when it contains but one element of the surface, see Fig. 255.

A *plane* is *tangent to a doubly curved surface* when it contains but one point of the surface, see Fig. 256.

A *point* at which tangency occurs is called a *point of tangency.*

A *line* along which two surfaces are tangent is called *a line of tangency.*

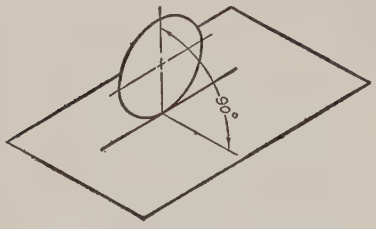


Fig. 254. A Plane Tangent to a Curve.

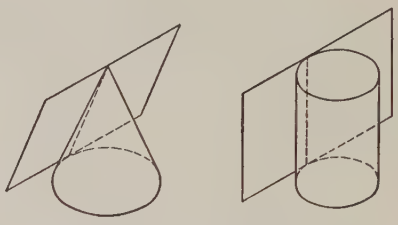


Fig. 255. Planes Tangent to Singly Curved Surfaces.

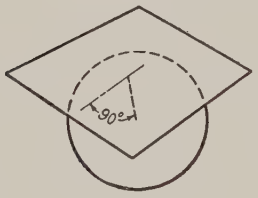


Fig. 256. A Plane Tangent to a Doubly Curved Surface.

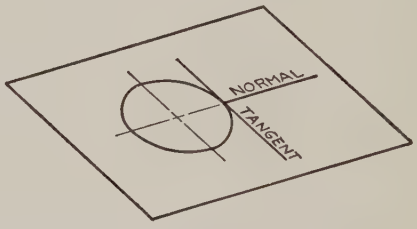


Fig. 257. A Straight Line Normal to a Curve.

Since a line of tangency must be an element of a singly curved surface, it is called an *element of tangency* in such cases.

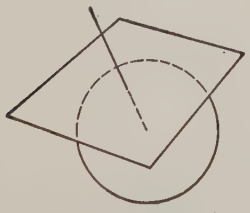


Fig. 258. A Straight Line Normal to a Doubly Curved Surface.

A *straight line* is said to be *normal to a plane curve* when it is in the plane of the curve and perpendicular to a tangent line at the point of tangency, see Fig. 257.

A *straight line* is *normal to a surface* when it is perpendicular to a tangent plane at the point of tangency, see Fig. 258.

72. Description of Tangent Lines. — With one exception, the graphic description of a pair of tangent lines shows

their representations to be tangent at the representation of the point of tangency, see Fig. 259. The exception occurs when the plane of the tangent lines is perpendicular to both

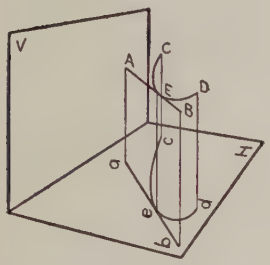


Fig. 259. Representations of Tangent Lines Are Tangent at the Representation of their Point of Tangency.

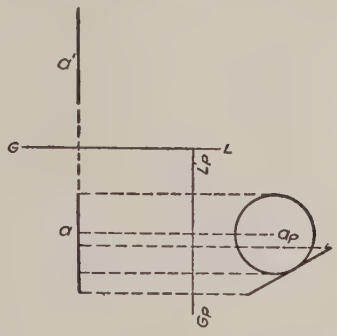
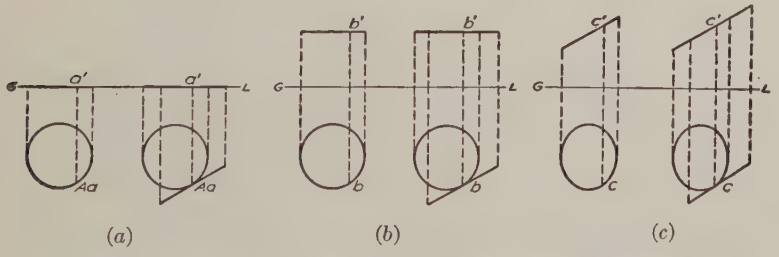


Fig. 260. Tangency Described by Employing an Additional Reference Plane.

reference planes. In such a case the point of tangency may be described by setting up a new pair of reference planes, see Fig. 260.

The direction of any representation of a straight line which is tangent to a plane curve is fixed by its tangent



(a) (b) (c)

Fig. 261. Descriptions of Straight Lines Tangent to Curves at given Points on the Curves.

relation to the corresponding representation of the curve. Hence, if one point on such a straight tangent line is given, the line may be immediately described, see Figs. 261 and 262.

These descriptions may be executed with somewhat

greater accuracy when the true shape of the curve is represented, that is, when the plane of the curve is parallel to, or coincident with, a reference plane. For this reason it is

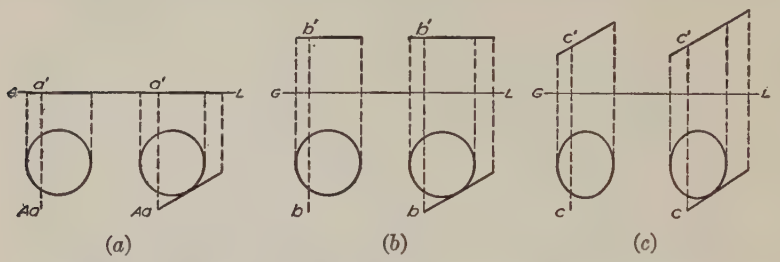


Fig. 262. Descriptions of Straight Lines Tangent to Curves and Containing Given Points not on the Curves.

sometimes desirable to add a new pair of reference planes, one of which is parallel to the given curve, see Fig. 263. Of course, the same result may be effected by rotating the curve about a properly established axis until the plane of the curve is parallel to, or coincident with, one of the primary reference planes; the tangent may be added in the rotated position and the description completed by counter-rotating the curve and tangent.

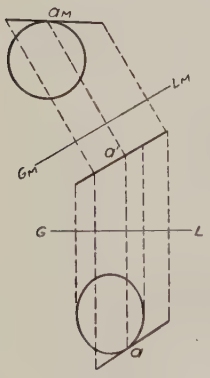


Fig. 263.

When the plane of a given curve is oblique to both primary reference planes, the primary description does not directly show that a given point is in the plane of the curve, see Fig. 264*a*. Since, by definition, a tangent cannot be drawn through a point not in the plane of a curve, data must be tested in this respect before attempting to establish tangents. The method of performing this test illustrated in Fig. 264*b* consists of first establishing the plane which contains the curve and then testing the point to see if it lies on a line of this plane. A shorter but sometimes less accurate test of the same order consists of checking the possibility

of drawing a cord or a diameter of the curve through the point. The test may also be performed by establishing a new pair of reference planes such that one reference plane is perpendicular to the plane of the curve. Much of the

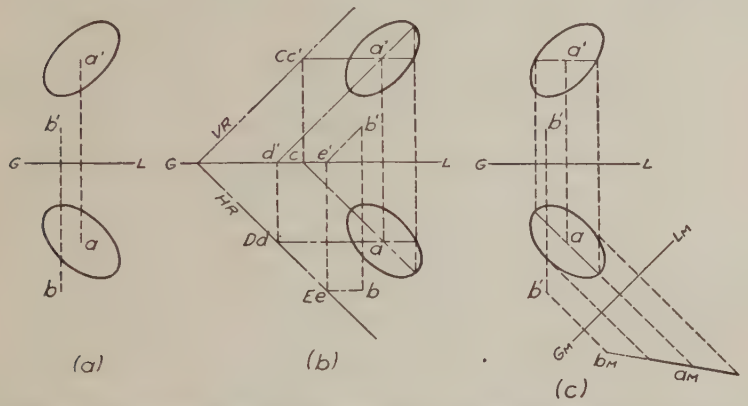


FIG. 264. Testing a Point to Determine if it Lies in the Plane of a Curve.

(*a*) The Given Point and Curve.

(*b*) The Point is Found to Lie in the Plane of the Curve Because it Lies on a Line of that Plane.

(*c*) The Point is Shown to Lie in the Plane of the Curve by the Representations of the Point and the Curve on a New Reference Plane Taken Perpendicular to the Plane of the Curve.

work of establishing the plane of the curve may be avoided in the case of a circle by taking the new ground line perpendicular to the major diameter of a representation of the circle, see Fig. 264*c*, as this diameter of the circle is parallel to a primary reference plane.

73. Description of Lines Tangent to Solids. — By definition a straight line is tangent to the curved surface of a solid when it is tangent to a line of that surface, see Fig. 253.

This fact suggests the method of attacking the problem of describing a straight line required to be tangent to the surface of a solid and to pass through a given point. The method consists of reducing the problem to the simpler one of describing a straight line required to be tangent to a plane curve and to pass through a given point. The simplifica-

tion is effected by passing a plane through the given point so as to cut a plane curve from the surface of the solid, see Fig. 265. Since the given point is in the plane of this curve, it is possible to establish a straight line through the

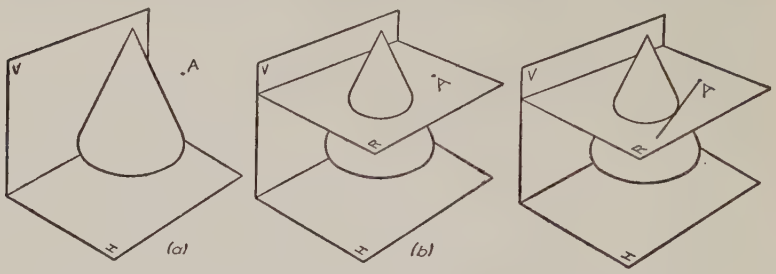


FIG. 265. Method of Establishing a Curve on the Surface of a Solid and in a Plane with a Given Point.

FIG. 266.

point and tangent to the curve, see Fig. 266. This line, by definition, is tangent to the surface of the solid.

When applying this method the draftsman should, for convenience sake, cut a *simple line* from the surface of the

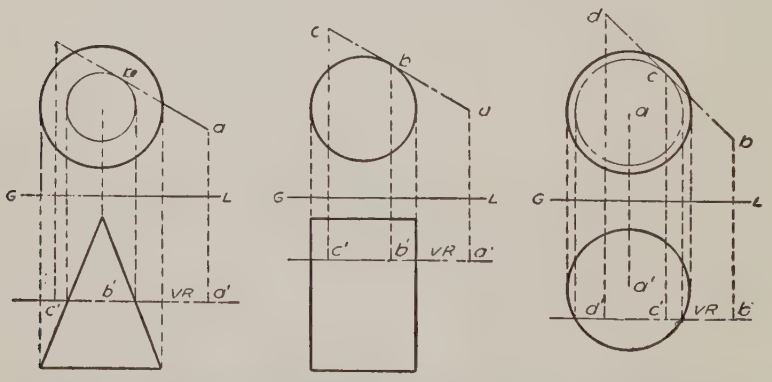


FIG. 267. Descriptions of Lines Tangent to Solids and Containing Given Points not on the Surfaces of the Solids.

solid in preference to a line more difficult to describe. Thus an auxiliary plane should be taken perpendicular to the axis of a surface of revolution when the axis is perpendicular

to a reference plane because simple circles are then cut from the surface, see Fig. 267. If the axis of a surface of revolution is not perpendicular to a primary reference plane a new pair of reference planes may be set up so as to establish this desirable arrangement, see Fig. 268.

The basic method of solving these problems is the same when the point is given on the surface of the solid as when given outside its surface. However, when the given point

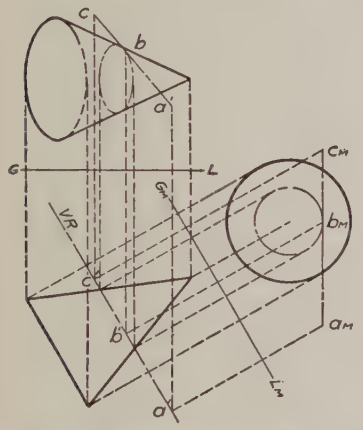


Fig. 268. Method of Employing an Additional Reference Plane when Describing a Line through a Given Point and Tangent to a Given Solid.

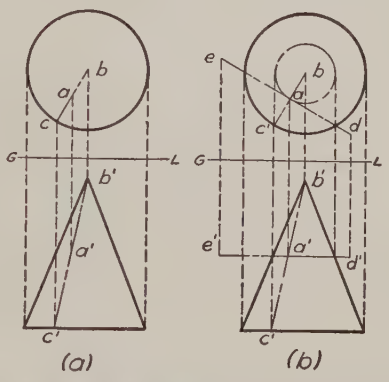


Fig. 269. The Method of Describing a Line Required to be Tangent to a Solid at a Given Point on its Surface.

is on a surface of revolution, a circle through the point may be readily established without using an auxiliary plane, providing the axis of the surface is perpendicular to a reference plane, see Fig. 269.

Since the sphere has an infinite number of axes of revolution its position relative to the primary reference planes is always favorable to the task of establishing simple circles on its surface.

74. Description of Planes Tangent to Solids — Preliminary Consideration. — Previous investigation of the general problem of describing planes in space has brought

out several facts with which the draftsman must be thoroughly familiar. Basic among these facts are those which relate to the fixation of a plane in space and to the method of graphically describing a plane.

Planes are preferably fixed in space by either of two methods: (1) by a pair of parallel or intersecting lines, (2) by a line of the plane and the direction of the plane; the direction being specified as perpendicular to a known line or plane, or as parallel to a known plane.

Planes are graphically described by those particular lines of the plane which are its traces on the reference planes.

Previous consideration has also made it clear that in order to describe a plane it is simply necessary to establish its traces. Although it is desirable to establish the traces of a required plane as directly as possible, the available data may not include a pair of intersecting or parallel lines nor a line and a direction which directly define the plane. The draftsman must therefore be capable of recognizing and employing that available information which is useful in establishing the defining data for the required plane.

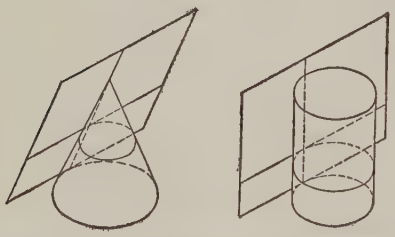


FIG. 270. The Element of Tangency and any Second Line which Intersects the Element and is Tangent to the Surface Define a Plane which is Tangent to the Surface.

75. Planes Tangent to Cones and to Cylinders. — Evidently the first step in devising a solution for the problem of describing a required tangent plane is to examine the given data with the view of detecting that information which will enable the defining data of the plane to be established.

By definition a plane is tangent to a singly curved surface when it contains but one element of that surface, see Fig. 255. An investigation of the relation of the tangent plane to the solid shows that the element of tangency and any line which intersects this element and is tangent to the

surface constitute a pair of intersecting lines which fix the position of the desired tangent plane, see Fig. 270.

ILLUSTRATIVE EXAMPLES OF PLANES TANGENT TO SINGLY CURVED SURFACES AT GIVEN POINTS ON THE SURFACES

Example 1. If a required tangent plane must contain a given point on the surface of the solid, the element of tangency is di-

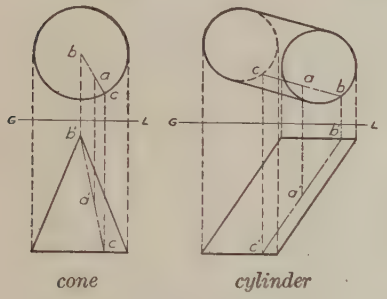


Fig. 271.

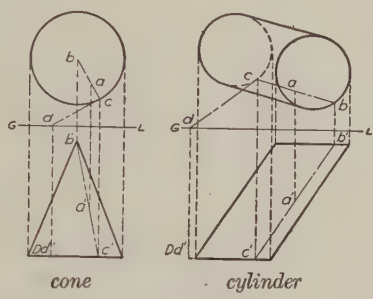


Fig. 272.

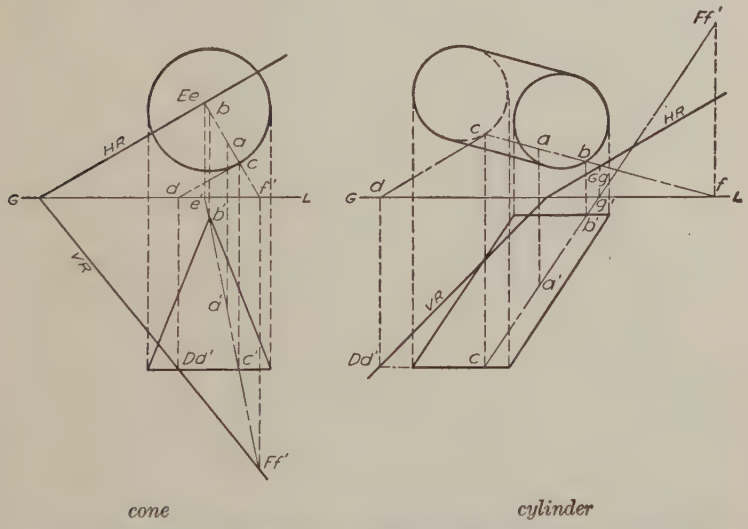


Fig. 273. Completed Descriptions of Planes Required to be Tangent to Singly Curved Surfaces and to Contain Given Points on the Surfaces.

rectly available, see Fig. 271. In such cases a pair of intersecting lines which define the required plane may be established by adding a second line which is tangent to the solid at any point along the

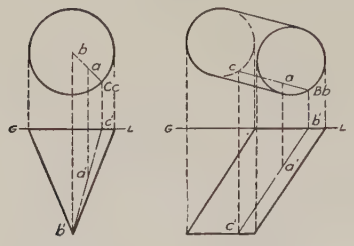


FIG. 274.

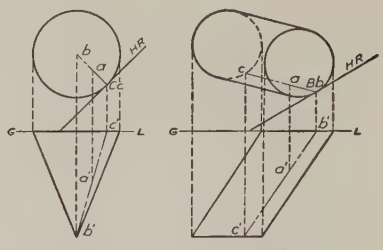


FIG. 275.

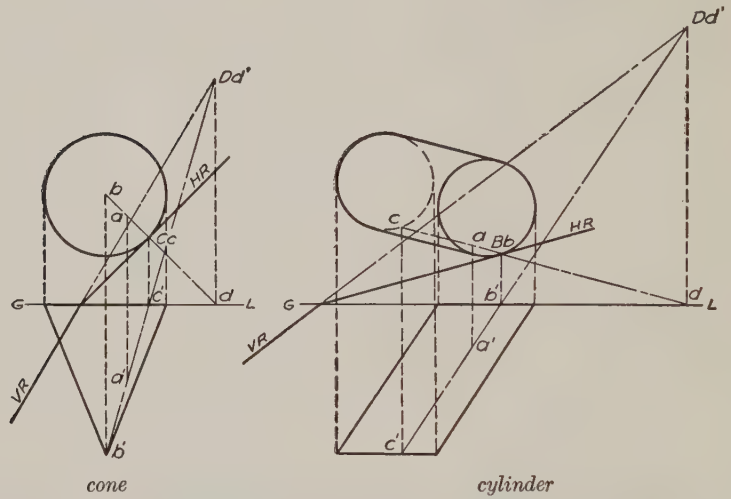


FIG. 276. Completed Descriptions of Planes Required to be Tangent to Singly Curved Surfaces and to Contain Given Points on the Surfaces. In these Descriptions a Trace Serves as a Defining Line of the Required Plane.

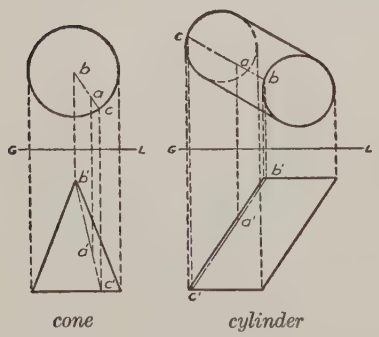


FIG. 277.

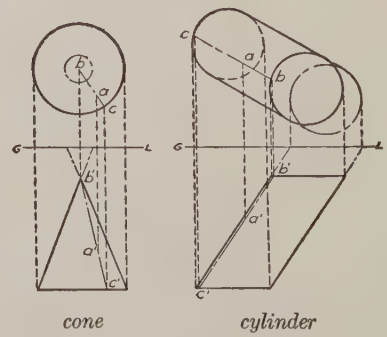


FIG. 278.

element of tangency. A base curve of the solid may be conveniently used when establishing this tangent line, see Fig. 272. The plane which contains the element of tangency and the tangent line is the desired tangent plane, see Fig. 273.

Example 2. When the given solid is so positioned that a base lies in a reference plane, see Fig. 274, the problem is somewhat simplified, because a line tangent to this base and intersecting the element of tangency is one trace of the required plane, see Fig. 275. The piercing point or the direction of the element of tangency completes the data which fix the other trace, see Fig. 276.

Example 3. The facts brought out in Example 2 may be applied to the solution of any problem in which the solid may be

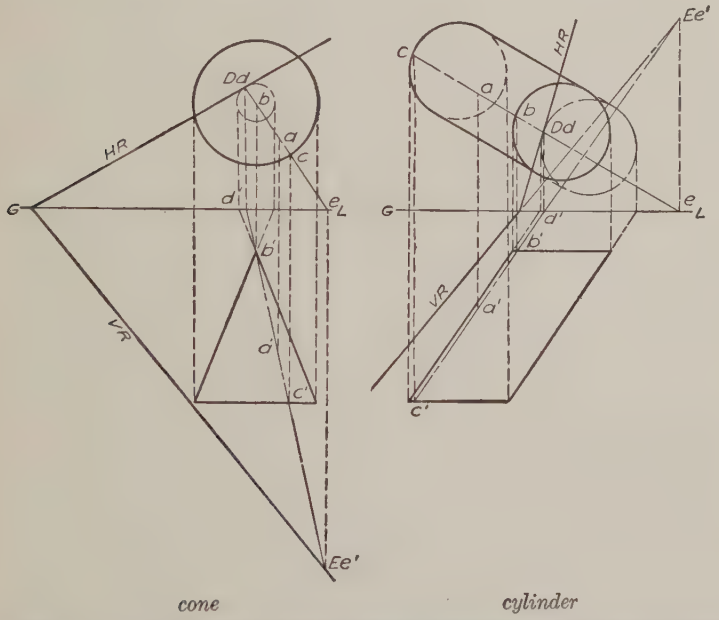


FIG. 279. Completed Descriptions of Planes Required to Be Tangent to Singly Curved Surfaces at Given Points on the Surfaces. In these Descriptions a Trace Serves as a Defining Line of the Plane.

extended until it meets a reference plane. Thus, in Fig. 277 both the cone and cylinder may be extended to a reference plane and the lines cut from these extended portions may be employed as were the base circles in Example 2, see Figs. 278 and 279.

Example 4. When a given solid is non-orthometrically positioned relative to the primary reference planes, see Fig. 280, it is still a simple matter to establish a plane tangent to the solid and through a given point on its surface.

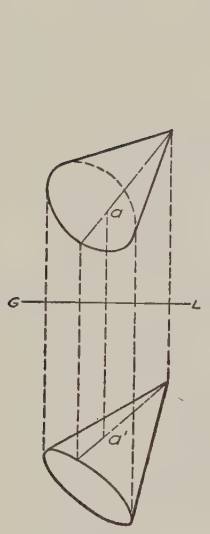


Fig. 280.

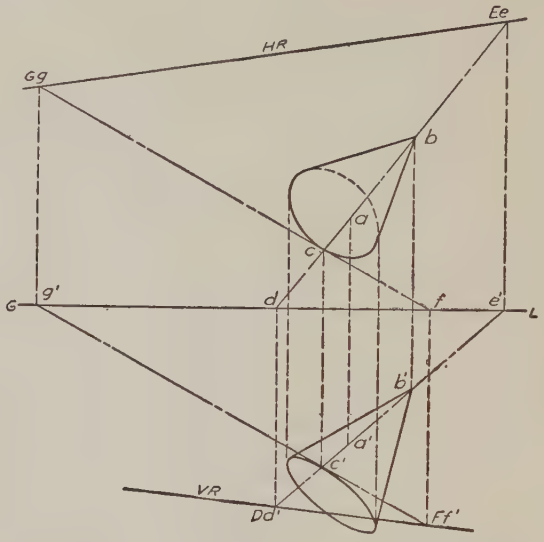


Fig. 281. A Completed Description of a Plane Tangent to a Non-Orthometrically Positioned Surface at a Given Point on the Surface.

The element of tangency and any line tangent to the solid and intersecting this element constitute a pair of intersecting lines which define the plane, see Fig. 281.

Of course such problems may always be reduced to a simpler form by establishing a new pair of reference planes orthometrically positioned relative to the given data.

ILLUSTRATIVE EXAMPLES OF PLANES TANGENT TO SINGLY CURVED SURFACES AND CONTAINING GIVEN POINTS NOT ON THE SURFACES

Example 1. If a required tangent plane must contain a given point not on the surface of the solid, the element of tangency is not directly available, see Fig. 282. However, a line through the point and tangent to the solid may be established by first establishing a proper curve on the solid, see Fig. 283. The point at

which this line is tangent to the solid supplies the information necessary to establish the element of tangency, see Fig. 284.

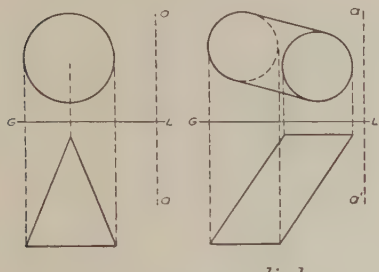


Fig. 282.

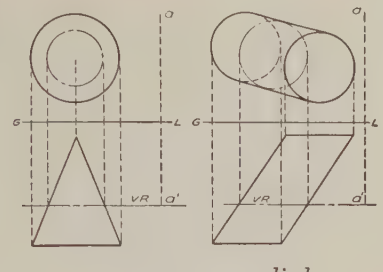


Fig. 283.

The pair of intersecting lines which define the required plane are now available and the description of the plane may be completed, see Fig. 285.

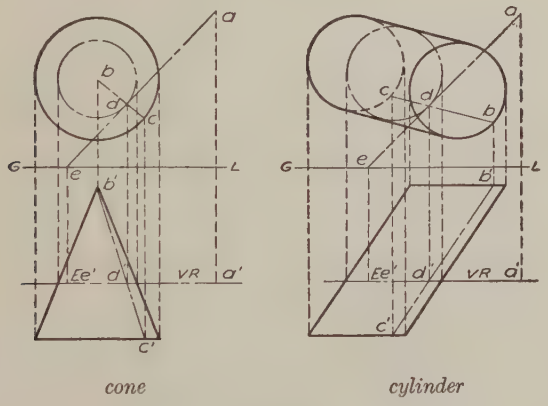
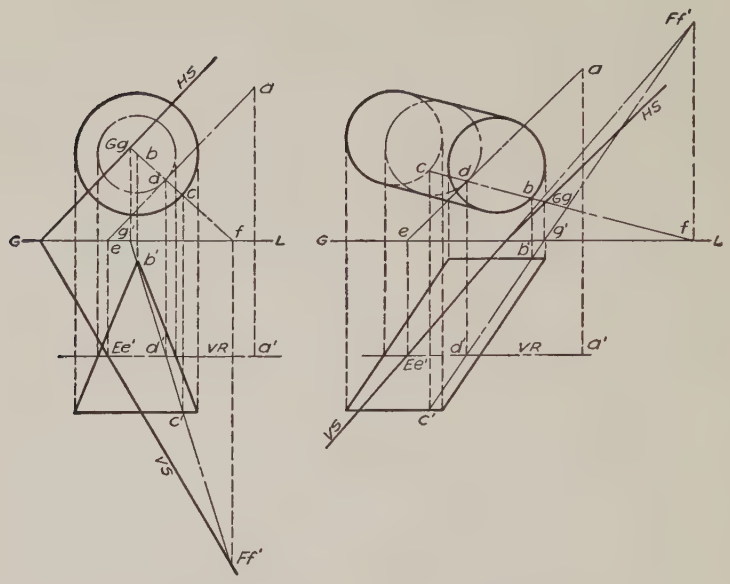


Fig. 284.

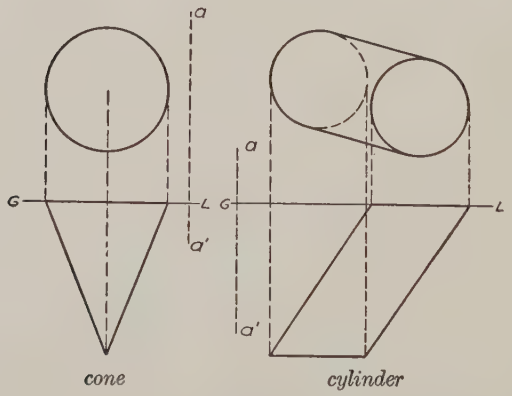
Example 2. The given data for this problem are shown in Fig. 286. Since a base of the solid lies in a reference plane, the trace of the required tangent plane in this reference plane could be immediately added if one point on the trace were available. The piercing point in this reference plane of any line of the required plane would supply the necessary point.

In the case of the cone it is known that all tangent planes must contain the apex of the cone. Hence a line through the apex and



cone *cylinder*

FIG. 285. Completed Descriptions of Planes Required to be Tangent to Singly Curved Surfaces and to Contain Points not on the Surfaces.



cone *cylinder*

FIG. 286.

the given point is a line of the required plane and the piercing points of this line may be used to establish the traces of the required plane, see Fig. 287.

In the case of the cylinder it is known that a line through the given point and parallel to the elements of the cylinder is a line of the required plane. The piercing points of this line supply the data necessary to locate the traces of the tangent plane, see Fig. 288.

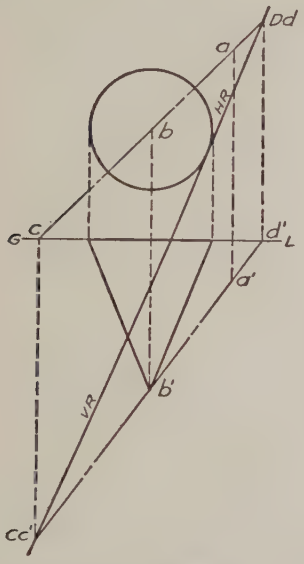


Fig. 287. A Completed Description of a Plane Required to Be Tangent to a Cone and to Contain a Given Point not on the Surface of the Cone.

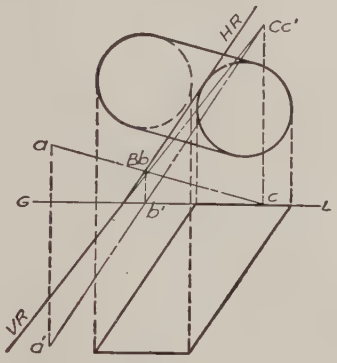


Fig. 288. A Completed Description of a Plane Required to be Tangent to a Cylinder and to Contain a Given Point not on the Surface of the Cylinder. The Plane in this Description Is Defined by Two Parallel Lines.

By extending a solid until it intersects a reference plane this scheme of solution may also be applied to a problem in which the given solid does not have a base in a reference plane.

76. Planes Tangent to Doubly Curved Surfaces at Given Points on the Surfaces. — A plane may be passed tangent to a solid having a doubly curved surface through a given

point on its surface, through a given point not on its surface, or through a line which does not intersect its surface.

By definition a plane is tangent to a doubly curved surface when it contains but one point of that surface. The plane must therefore be perpendicular to a radius of the surface at the point of tangency. This fact is the basis of a scheme for establishing planes tangent to such surfaces as spheres and tori at given points on their surfaces, because the given point and the direction of the radius at that point supply the necessary defining data for the required plane.

ILLUSTRATIVE EXAMPLES

Example 1. The given data for this example consist of a sphere and a point on the surface of the sphere, see Fig. 289. In accordance with the above discussion a plane which contains the

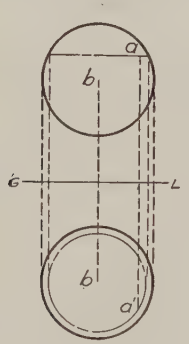


Fig. 289.

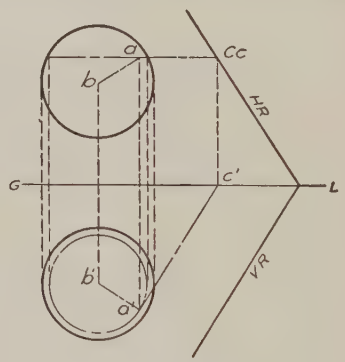


Fig. 290. A Completed Description of a Plane Required to be Tangent to a Sphere at a Given Point on its Surface.

given point and is perpendicular to the radius of the sphere at the point is tangent to the sphere. Evidently the problem is simply to pass a plane through a given point and perpendicular to a given line, see Fig. 290.

Example 2. Let it be required to describe a plane which is tangent to the torus, Fig. 291, at a given point on its surface. The problem is essentially the same as that in Example 1, in that a

plane must be established through a given point and perpendicular to a line. The line in this case is radius of the generating circle of the torus, see Fig. 292. Fig. 293 shows the completed description.

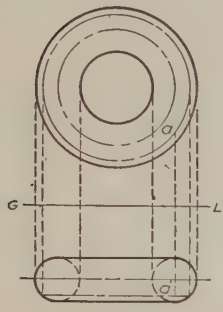


Fig. 291.

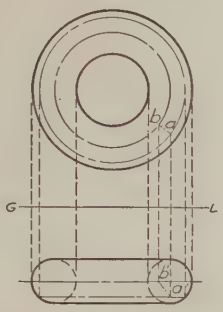


Fig. 292.

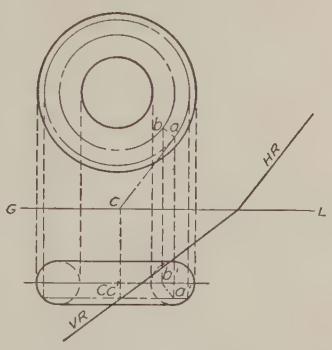


Fig. 293. A Completed Description of a Plane Required to Be Tangent to a Torus at a Given Point on its Surface.

77. Planes Tangent to Doubly Curved Surfaces and Containing Given Points not on the Surfaces. — A plane which contains a given point outside a doubly curved surface and is tangent to that surface will contain a line through the given point and the point of tangency. Hence, if a line is passed through the given point and tangent to the given solid, the problem is reduced to that of passing a plane tangent to the solid at a point on its surface, the point being the point of tangency of the line. Inasmuch as an infinite number of lines may be passed through a given point tangent to a doubly curved surface, an infinite number of planes may also be passed through the point tangent to the surface.

ILLUSTRATIVE EXAMPLES

Example 1. Let it be required to pass a plane tangent to a sphere, Fig. 294, and through a given point not on the surface of the sphere.

In accordance with the preceding discussion it is first necessary

to establish a line through the point and tangent to the sphere, see Fig. 295. A plane tangent to the sphere at the point of tangency of this line contains the line, hence the given point, see Fig. 296.

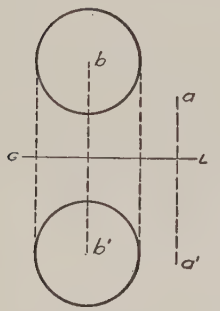


Fig. 294.

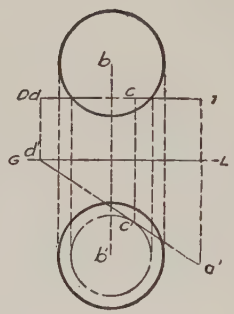


Fig. 295.

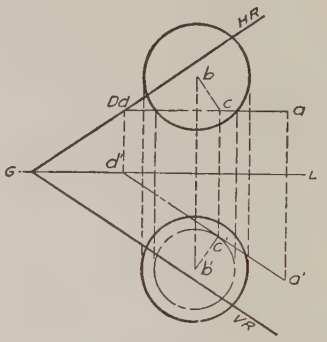


Fig. 296. A Completed Description of a Plane Required to be Tangent to a Sphere and to Contain a Given Point not on the Surface of the Sphere.

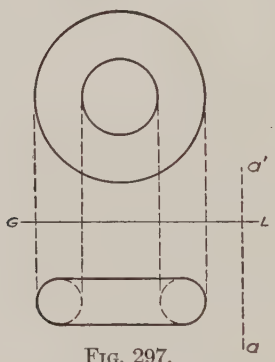


Fig. 297.

Example 2. Let it be required to pass a plane tangent to a torus and to contain a given point, see Fig. 297. As in Example 1 a tangent line is first established, see Fig. 298, and the required plane is passed tangent to the torus at the point of tangency of this line, Fig. 299.

78. Planes Tangent to Doubly Curved Surfaces and Containing Given Lines. — Two planes may be passed through a given line and tangent to a given surface of revolution.

Since one defining line of the required plane is given, a

solution may be effected by establishing a second line. Any line through the point of tangency and intersecting the given line may be employed as this second defining line.

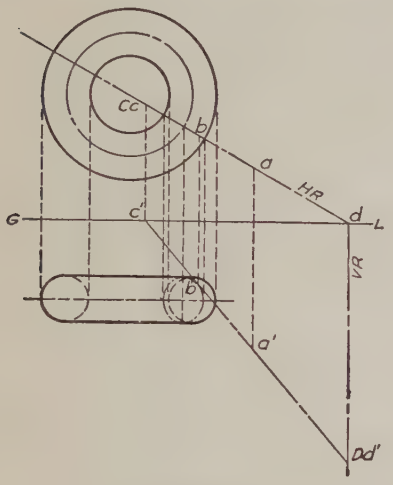


Fig. 298.

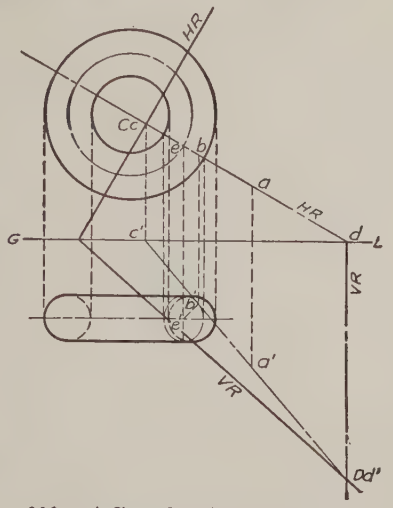


Fig. 299. A Completed Description of a Plane Required to Be Tangent to a Torus and to Contain a Given Point not on the Surface of the Torus.

ILLUSTRATIVE EXAMPLES

Example 1. Let it be required to pass a plane through a given line and tangent to a given sphere, see Fig. 300.

Upon examining the relation of such a tangent plane to the sphere it may be seen that any line which intersects the given line and which passes through the point of tangency is tangent to a great circle of the sphere, and is therefore contained by the plane of the great circle, see Fig. 301. Furthermore, it may be seen that, in every case, the plane of the great circle is perpendicular to the required tangent plane, and that in one case the plane of the great circle is perpendicular to the given line.

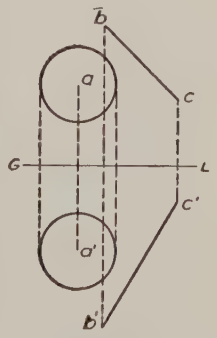


Fig. 300.

If a plane is passed through the center of the sphere and perpendicular to the given line it will cut a great circle from the sphere and a point from the given line. A line through this point and tangent to the great circle is a line of the required tangent plane, see Fig. 302.

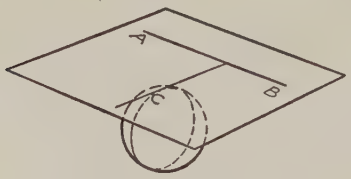


Fig. 301.

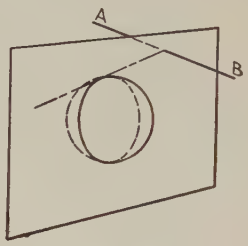


Fig. 302.

When applying this scheme a more accurate result may be reached and considerable drafting work may be avoided if the auxiliary plane, which cuts the great circle from the sphere and

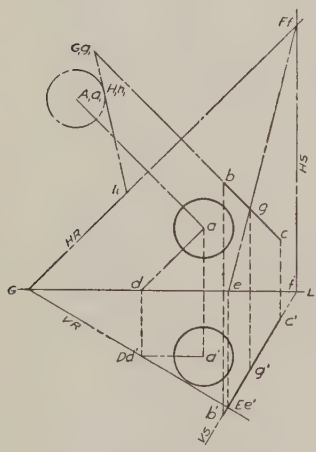


Fig. 303.

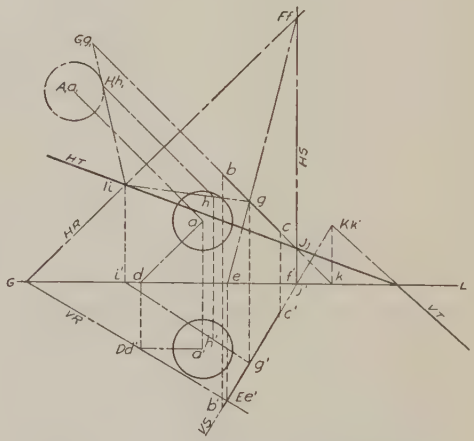


Fig. 304. A Completed Description of a Plane Required to be Tangent to a Sphere and to Contain a Given Line.

the point from the given line, is rotated about a trace into a reference plane. The desired line through the point and tangent to the great circle may be readily added while the point and circle are in this rotated position, see Fig. 303. A counter-rotation

establishes the line in its true position and makes it available as a defining line of the required tangent plane, see Fig. 304.

Example 2. Let it be required to pass a plane through a given line and tangent to a given torus, see Fig. 305.

An examination of the relation of the tangent plane to the torus shows that, as for the sphere in the preceding problem, any line which passes through the point of tangency and which intersects

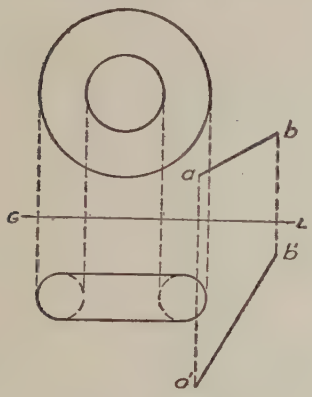


Fig. 305.

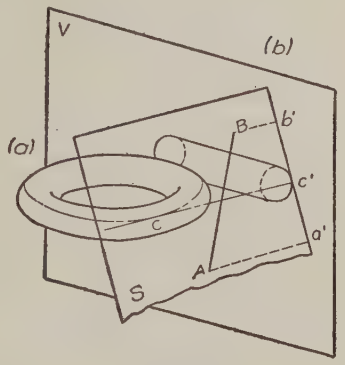


Fig. 306.

the given line would with the given line determine the position of the required tangent plane. If the procedure used in the preceding problem for establishing such a line is attempted it is found that the solution is both long and inaccurate due to the amount of delineation work involved in determining the section cut from the torus by the auxiliary plane. The following method although incorporating a trial and error operation is more accurate and shorter.

An examination of the relation of the required plane to the torus reveals that one line of the required plane is tangent to a transverse circle on the surface of the torus, see Fig. 306*a*. The point at which this line intersects the given line is represented coincidently with the point of tangency upon a reference plane parallel to the axis of the torus and perpendicular to the tangent plane, see Fig. 306*b*. If an existing reference plane is parallel to the axis of the torus, this desirable arrangement of the given data relative to that reference plane may be established by a cut-and-try rotation, which consists of rotating the given line about the axis

of the torus until the representation of the line is tangent to the representation of the torus, see Fig. 307. After establishing the

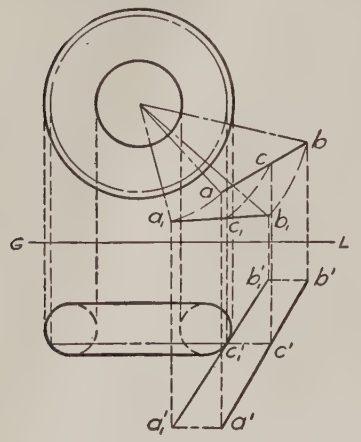


Fig. 307.

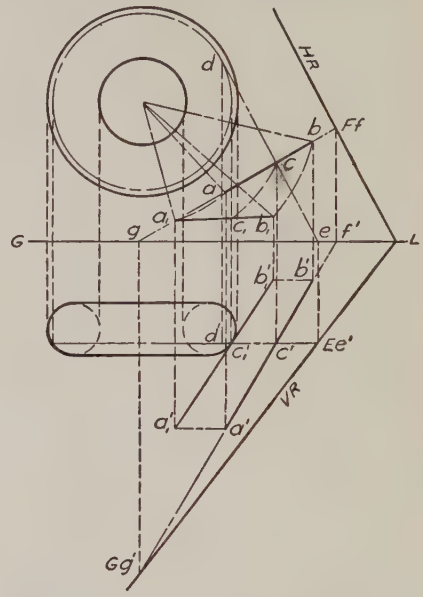


Fig. 308. A Completed Description of a Plane Required to be Tangent to a Torus and to Contain Any Given Line.

point of tangency in its normal position, by a counter-rotation, the data for defining the required tangent plane are available and the description of the plane may be completed, see Fig. 308.

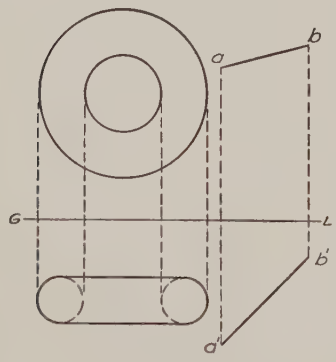


Fig. 309.

Example 3. If the given line intersects, or is parallel to the axis of the torus, see Fig. 309, the task of determining the point of tangency is more simple. In these special cases the given line generates a circular cone or cylinder as it is revolved about the axis of the torus, hence the representation of the line which is tangent to the torus may be drawn without resorting to cut-and-try methods, see Fig. 310.

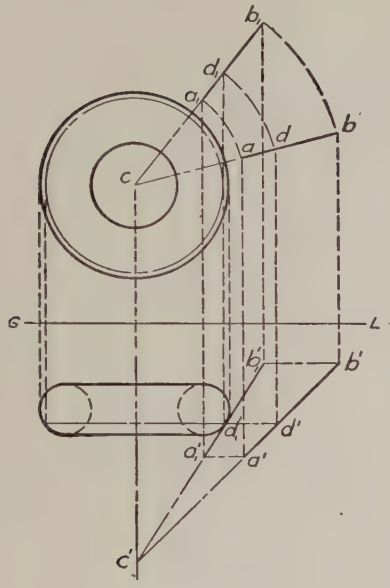


FIG. 310.

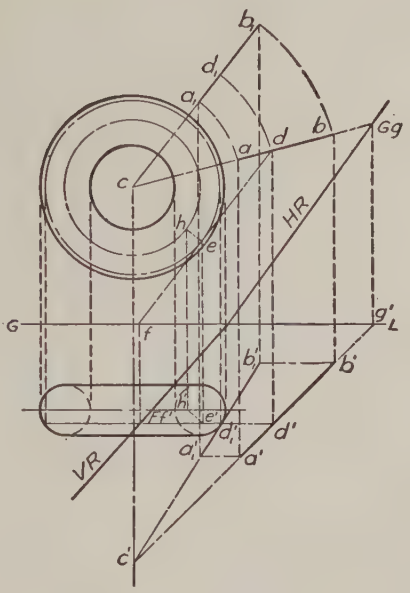


FIG. 311. A Completed Description of a Plane Required to Be Tangent to a Torus and to Contain a Line which Intersects the Axis of the Torus.

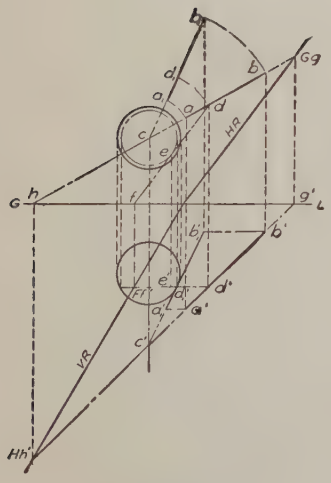


FIG. 312.

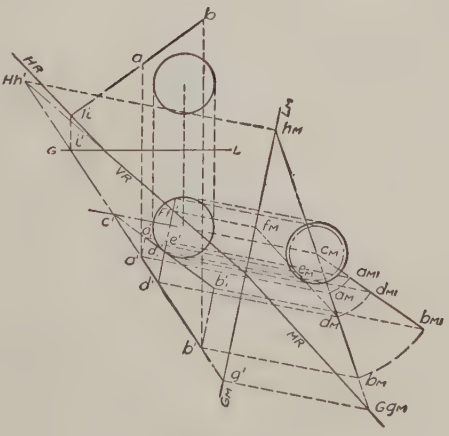


FIG. 313.

The completed description of the tangent plane for this example is shown in Fig. 311.

The general method employed for solving the above problems may be applied to the solution of any problem in which a plane is required to pass through a given line and be tangent to a given doubly curved surface of revolution. Problems of this nature which involve spheres are all of the simple type as, in all cases, an axis may be assumed through the sphere so as to intersect the given line. Figs. 312 and 313 illustrate respectively solutions for problems in which this axis can and can not be taken perpendicular to a primary reference plane.

CHAPTER X

DESCRIPTION OF LINES, PLANES, PLANE FIGURES AND SOLIDS IN SPECIFIED POSITIONS RELATIVE TO REFERENCE PLANES

79. Introduction. — Lines have but one dimension, that of length. Planes have both length and width. Because of these dimensions a line and a plane have, in addition to the *positions* of their points, the property of *direction*. To be fully versed in the elements of graphic description a draftsman must be able to describe both lines and planes in accordance with specified directions relative to the reference planes.

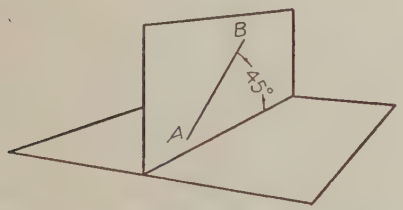


FIG. 314. The Angle between a Line and a Plane Is Measured in a Second Plane, Perpendicular to the First and Containing the Line.

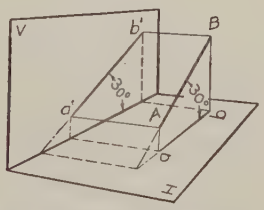


FIG. 315. When a Line Is Parallel to a Reference Plane its Representation on that Plane Makes the Same Angle with the Ground Line as the Line Makes with the Mate of the Reference Plane.

The direction of a line or plane relative to a reference plane is measured by the angle between the line or plane and the reference plane. The angle between a line and a plane is measured in a second plane which passes through the line and is perpendicular to the first, see Fig. 314. If a line in space is parallel to one of a pair of reference planes, the angle between the representation of the line on this reference plane and the ground line is a true value repre-

sentation of the actual angle between the line in space and the mating reference plane. Thus, in Fig. 315 the line AB is parallel to V, hence the true angle between AB and H is represented on the V-plane.

The angle between two planes is measured in a third plane which is perpendicular to both the given planes. The value of the angle is measured between the two lines which are cut, one from each of the given planes, by the third

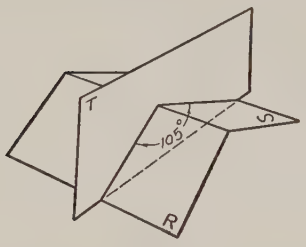


Fig. 316. The Angle between Two Planes Is Measured in a Third Plane which Is Perpendicular to Both.

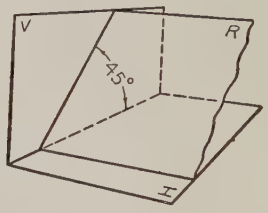


Fig. 317. When a Plane in Space Is Perpendicular to One of a Pair of Reference Planes its Trace on that Reference Plane Makes the Same Angle with the Ground Line as the Plane Makes with the Other Reference Plane.

plane, see Fig. 316. Hence, if a plane in space is perpendicular to one of a pair of reference planes, the angle between its trace on that reference plane and the ground line is the actual angle between the plane in space and the mating reference plane, see Fig. 317.

80. To Describe a Line which Makes a Specified Angle with One Reference Plane. — It is a property of a right circular cone that all elements are at equal angles with the plane of its base. Hence, if the axis of a right circular cone is perpendicular to a reference plane, all elements of the cone are at equal angles with that reference plane. This fact enables a draftsman to describe readily a line that is required to make a given angle with one of the primary reference planes and to be oblique to the second.

Example. Let it be required to describe a line 3 inches in length with one end at a given point A, and which makes an angle of 45° with the H-plane and is oblique to V.

First the line is described while parallel to V as in this position the true angle between the line and the H-plane is shown between the V-representation of the line and the ground line. Since the true length of the line is also shown by the V-representation when the line is parallel to V, this representation is drawn to the specified length, see Fig. 318. If the line is now revolved about an axis through A and perpendicular to H, a right circular cone is generated every element of which is at 45° with the H-plane, see Fig. 319. Any element other than the two which are parallel to V may be selected for the required line, see Fig. 320.

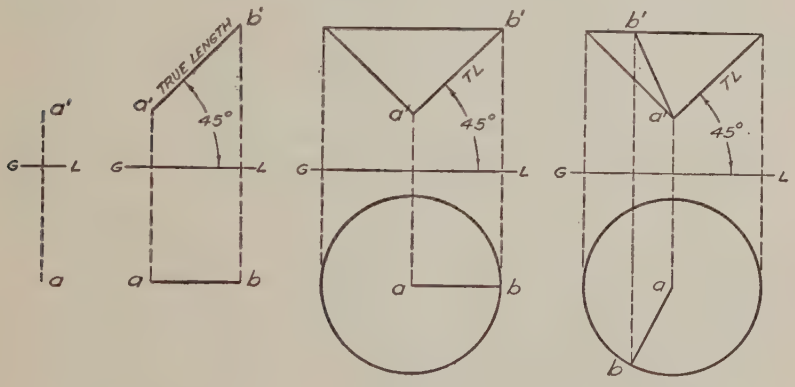


Fig. 318. Fig. 319. Fig. 320.

81. To Describe a Line which Makes a Specified Angle with Each of a Pair of Reference Planes. — A line which serves as a common element of two right circular cones, one with axis perpendicular to H and one with axis perpendicular to V, must make angles with the V- and H-reference planes which are equal respectively to those made by the elements of the cones, see Fig. 321. This fact is the basis for a method of establishing a line at specified angles to both of a pair of reference planes. It should be noted, however, that the sum of the values which may be assigned to the angles of such a line cannot exceed 90° because

when the sum is greater, the cones cannot have a common element, see Fig. 322.

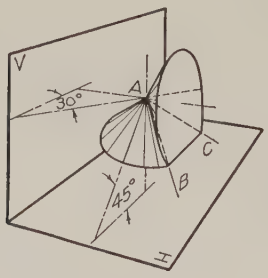


Fig. 321.

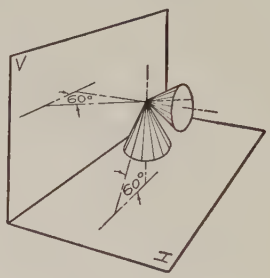


Fig. 322.

Example. Let it be required to describe a line 3 inches long which has one end at a given point A and which makes 30° with the H-plane and 45° with the V-plane.

The first step is to describe one of the construction cones, say the one whose axis is perpendicular to H and whose elements make

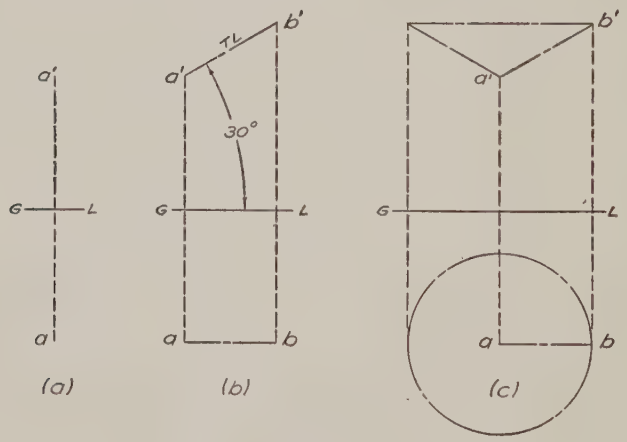


Fig. 323.

30° with the H-plane, see Fig. 323. When describing this cone one of the two elements which are parallel to V is represented first, as the V-representation of this element shows the true length of the line and also the true angle between the line and the H-plane, see Fig. 323*b*. By assuming an axis through the given point A

and perpendicular to H the desired construction cone may be generated, see Fig. 323*c*.

Next, describe the second construction cone, the axis of which must be perpendicular to V and whose elements must make 45° with V. To describe this cone first represent one of the two elements which are parallel to H, as the H-representation of this element will show the true length of the line and also the true angle between the line and the V-plane, see Fig. 324*a*. By establishing an axis through the point *A* and perpendicular to *V* the second construction cone may be generated, see Fig. 324*b*.

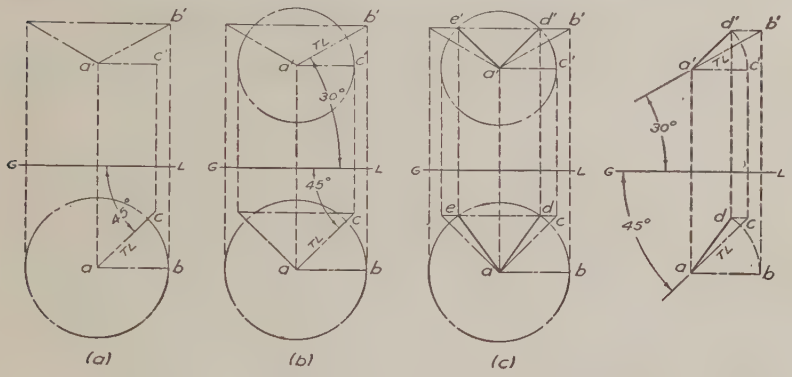


FIG. 324.

FIG. 325. A Practical Description of a Line which Makes a Specified Angle with each of the Primary Reference Planes.

With both construction cones described the representations of the points common to the bases of the cones may be recognized. These points and the common apex determine the common elements of the cones. Two such points are found to exist, as the sum of the specified angles is less than 90°, hence there are two lines which meet the requirements, see Fig. 324*c*. Since there are two other lines on the surface of the vertical cone diametrically opposite those already described, which also meet the requirements of the problem, there are four such lines in all.

Fig. 325 illustrates a practical drawing for describing a line which makes specified angles with the reference planes; note the absence of unnecessary delineation.

82. To Describe a Plane which Makes a Specified Angle with One Reference Plane. — The true value of the angle between a plane in space and a reference plane is directly described only when the plane is perpendicular to a mate of the reference plane, see Fig. 317. However, a plane may be established in space so as to make a specified angle with one reference plane and to be oblique to its mate by establishing the plane tangent to a right circular cone whose elements make the desired angle with the reference plane, see Fig. 326.

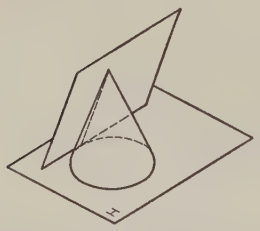


FIG. 326. If a Plane in Space is Tangent to a Cone whose Axis is Perpendicular to a Reference Plane, the Plane Makes the Same Angle with the Reference Plane as the Elements of the Cone.

Example. Let it be required to describe a plane in space which will contain a given line AB and make 45° with the H-plane. The foregoing discussion suggests the method of solution. The required plane may be established by passing it through

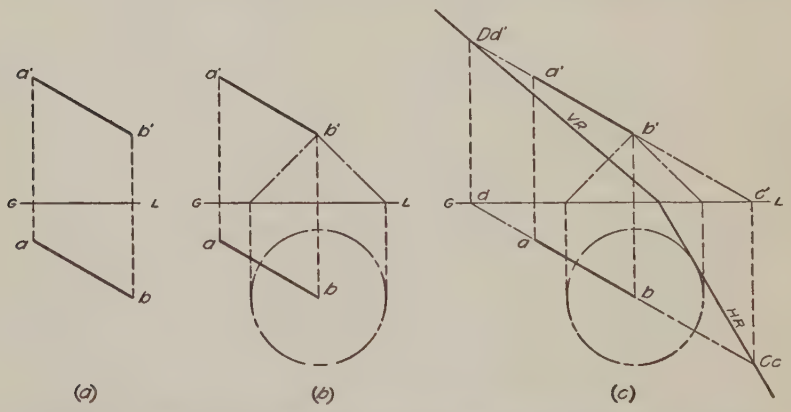


FIG. 327. A Method of Describing a Plane which Is Required to Contain a Given Line and to Make a Specified Angle with a Reference Plane.

the given line and tangent to a 45° right circular cone whose axis is perpendicular to H.

The execution of the solution is simplified if the apex of the cone

is assumed on the given line and if the base of the cone is taken in the H-plane, see Fig. 327*b*. With the cone assumed in this manner the H-trace of the required plane is fixed by the H-piercing point of the given line and by the direction necessary for the trace to be tangent to the base circle of the cone. The V-trace is fixed by two points, as it meets the H-trace at the ground line and passes through the V-piercing point of the given line, see Fig. 327*c*.

83. To Describe a Plane which Makes a Specified Angle with Each of a Pair of Reference Planes. — Since the angle made by a plane in space with a reference plane is the complement of the angle made by a line which is perpendicular to the plane, see Fig. 328, it is apparent that the sum of the angles a plane may make with a pair of reference planes is limited to the sum of the complements of the angles a line may make. The sum of the angles an oblique line may make with a pair of reference planes cannot exceed 90° as explained in Art. 81; hence, the sum of the angles an oblique plane in space may make with the reference planes cannot be less than 90°.

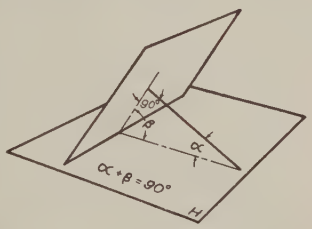


Fig. 328. If a Line and a Plane Are Perpendicular in Space, they Make Complementary Angles with a Reference Plane.

Further comparison of the plane problem with the line problem shows that, since there are four positions of a line which satisfy the requirements, there are also four positions of a plane which meet the requirement of making specified angles with the reference planes.

The above discussion suggests a scheme for determining the description of a plane which must pass through a given point and make specified angles with the reference planes. This scheme consists of first establishing a line which makes angles with the reference planes equal to the complements of the angles desired of the plane. The plane is then passed through the given point perpendicular to the line.

Example. Let it be required to describe a plane which contains a given point A and which makes 45° with H and 60° with V.

First, at any convenient place on the drawing, a line is described which makes 45° with H and 30° with V, see Fig. 329. In this example the common apex of the two construction cones is assumed on the ground line as the construction may thus be performed more rapidly.

Next the required plane is described so as to contain the given point and to be perpendicular to the complementary line, see Fig. 330.

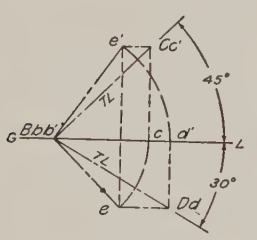


Fig. 329.

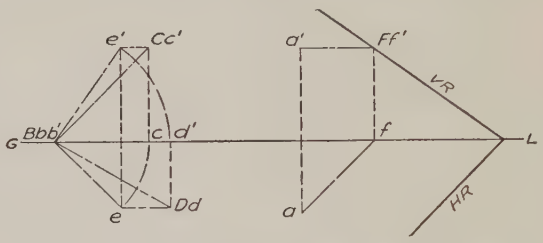


Fig. 330. A Method of Describing a Plane in Space which Is Required to Contain a Given Point and to Make a Specified Angle with Each of the Primary Reference Planes.

84. To Describe a Plane Figure which Makes a Specified Angle with One of a Pair of Reference Planes and is Perpendicular to the Other. — This problem is solved by locating the plane figure in a plane in space which has the desired arrangement with the reference planes. The first step of the solution is to describe the plane in space which bears the plane figure. If this plane is then rotated about one of its traces into a reference plane, the true representation of the required plane figure may be added in this rotated position. By counter-rotating the plane to its original position the representation of the figure in the required position may be established.

Example. Let it be required to describe a 2-inch square which has for its center a given point A and the plane of which makes an angle of 45° with H and is perpendicular to V.

The plane of the figure is first established, in this case making 45° with H and perpendicular to V and containing the given point

A, see Fig. 331*a*. This plane is then rotated into H about its H-trace, see Fig. 331*b*.

This drawing, Fig. 331*b*, also illustrates a simple application of a very practical method of rotating a point with the plane in which it lies. The scheme consists of establishing a line through the point and parallel to that trace of the plane which serves as the

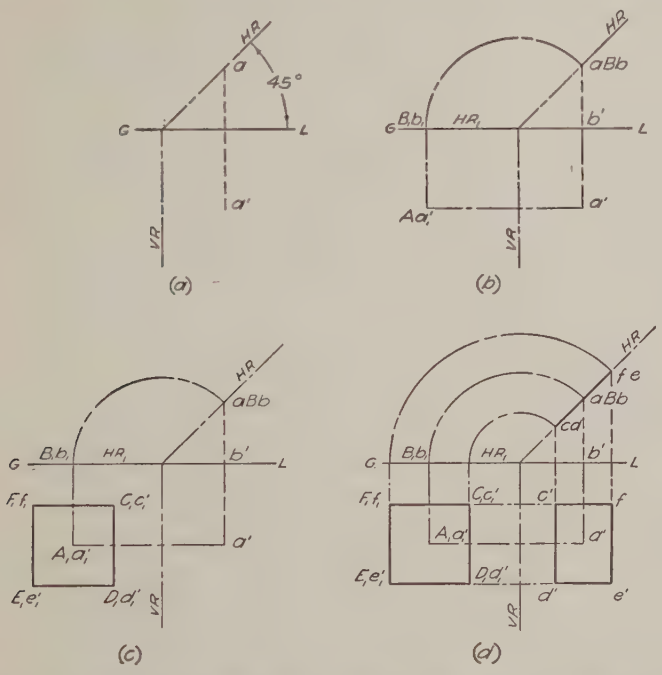


FIG. 331. A Method of Describing a Plane Figure which Is Required to Be Perpendicular to One Primary Reference Plane and to Make a Specified Angle with the Other.

axis of rotation. The single piercing point of this line is on the other trace of the plane. The rotated position of the piercing point is easily established and, since the line remains parallel to the axis during rotation, the rotated position of the line may also be described. Since the required point is on this line it may now be described in its rotated position.

The next step is to represent the given plane figure in the rotated position, see Fig. 331*c*. It is possible during this operation to

establish the figure so that its sides make desired angles with either of the traces of its plane.

The last step consists of counter-rotating the plane and plane figure to the original position of the plane. Points on the plane figure are treated in exactly the same manner as the center point. Lines parallel to the trace which serves as the axis are established through the points; the piercing points of these lines are found and the lines described in their counter-rotated positions by the point and direction method. The representations of the points of the figure are then located on the corresponding representations of the lines which bear them, see Fig. 331*d*.

85. To Describe a Plane Figure in Any Specified Position Relative to a Pair of Reference Planes. — The procedure followed in solving the preceding problem may be applied to all problems in describing plane figures in specified positions. These problems differ only in the method employed for establishing the plane of the required figure.

Example 1. Let it be required to describe a hexagon, 2 inches across flats, with its center at a given point A and making angles of 60° with H and 45° with V. One side of the hexagon is parallel to the H-plane.

The first step is to establish the plane of the hexagon. This is accomplished by the method explained in Art. 83, see Fig. 332*a*.

Next the plane of the figure and the given point are rotated into a reference plane about a trace of the plane as an axis. The rotated representation of the hexagon is then added, care being taken to make one side parallel to the H-trace of the plane in accordance with the specifications, see Fig. 332*b*. Lines are assumed through the corner points of the hexagon and parallel to the trace which serves as the axis, see Fig. 332*c*. The representations of the counter-rotated positions of these lines and of the corner points are then added and the drawing completed, see Fig. 332*d*.

Example 2. Let it be required to describe the hexagon of the previous problem but with one side of the hexagon making 45° with the H-plane instead of being parallel.

As in the preceding problem the plane of the hexagon is first established so as to contain the given point, see Fig. 333*a*.

The next step, however, consists of establishing a construction

line in this plane which makes an angle of 45° with H. This is accomplished by establishing a 45° right circular cone with base in the H-plane and its apex on a line of the plane. Either of the

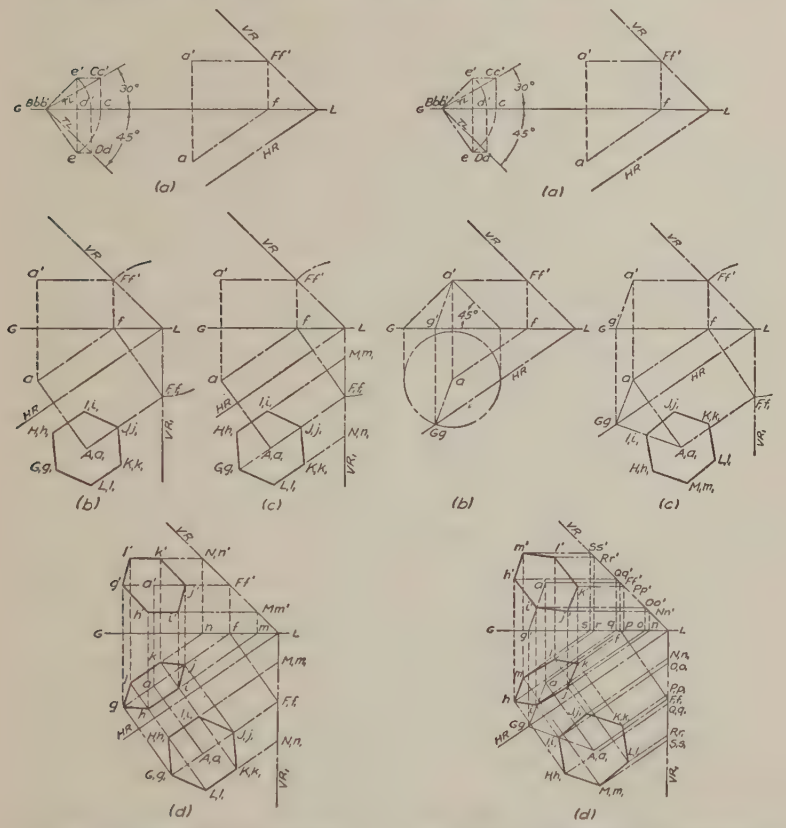


Fig. 332. A Method of Describing a Plane Figure which Is Required to Make a Specified Angle with Each of the Primary Reference Planes.

Fig. 333. A Method of Describing a Plane Figure which Is Required to Make a Specified Angle with each of the Primary Reference Planes and One Edge of which Is Required to Make a Specified Angle with a Reference Plane.

two elements cut from this cone by the plane may be used as the required construction line, see Fig. 333*b*.

The plane of the figure is now revolved into a reference plane and the revolved representation of the hexagon added, care being

taken to make one side parallel to the revolved representation of the construction line, see Fig. 333*c*. A counter-rotation completes the problem, see Fig. 333*d*.

86. To Describe a Solid in a Specified Position Relative to a Pair of Reference Planes. — The position of a solid relative to a pair of reference planes may be specified by stating the position of some point on the solid and the angles made by some line of the solid with the reference planes. The following example illustrates the procedure of determining the description of a solid when its position is specified in this manner.

Example. Let it be required to describe a right pyramid in the third angle with its apex 1 inch from H and 2 inches from V and its axis making angles of 60° with H and 15° with V. The pyramid has a 2-inch square base and its altitude is $2\frac{1}{2}$ inches.

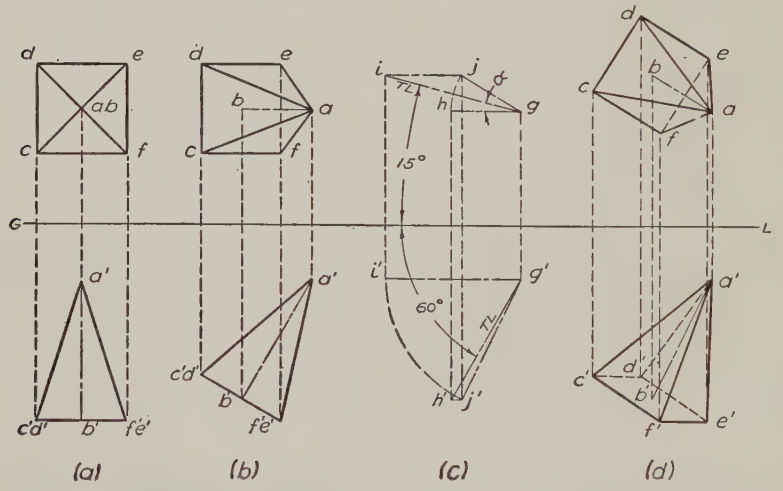


Fig. 334. A Method of Describing a Solid whose Axis Is Required to Make a Specified Angle with Each of the Primary Reference Planes.

The procedure of the solution is first to describe the solid in an orthometric position and then by two successive rotations to change its position to that which is specified.

Fig. 334*a* illustrates the description of the pyramid in the orthometric position with its apex in the position specified.

The next step of the solution is to rotate the pyramid about an assumed axis perpendicular to V and through the apex, until the axis of the solid makes the required angle (60°) with H, see Fig. 334*b*.

If the pyramid is now rotated about an axis through the apex and perpendicular to H, the axis of the solid would generate a right circular cone all elements of which would make angles of 60° with H and some elements of which would make the desired angle of 15° with V. The angle through which the pyramid must be rotated in order to have its axis at an angle of 15° with V is not known. However, it is known that the desired representations of the axis of the pyramid will be parallel to the corresponding representations of a line which makes the required angles with V and H. This fact suggests the scheme of establishing a construction line (using the double cone method explained in Art. 81) which makes angles of 60° with H and 15° with V. The angle through which the pyramid must be rotated is that necessary to bring the H-representation of its axis parallel to the H-representation of this construction line, see Fig. 334*c*.

The completed description of the solid in the position specified is shown in Fig. 334*d*.

CHAPTER XI

ELEMENTS OF GRAPHIC COMPUTATION

87. Summary of the Study of Graphic Description. — Upon completing a study of the preceding chapters the student should be satisfied that he thoroughly understands the basic system of graphic description and that he has acquired some facility in the application of this system to the description of those in-space relations of points, lines, planes, and solids which have been given consideration. If these objectives have been gained the student is prepared to undertake the task of describing any in-space arrangement of points, lines, planes, and solids, regardless of the complexity of their arrangement and regardless of the manner in which information concerning their relative positions may be specified.

Although it is possible, and in many cases advisable, to learn by rote the solutions of certain frequently encountered problems, it is quite impossible to acquire a full working knowledge of the subject in this manner, as problems occur in an infinite variety. Each new problem must be solved by breaking it up into a sequence of those lesser problems whose solutions are already thoroughly understood. As the draftsman acquires greater experience he becomes familiar with the solutions of more and more problems. The need of making detailed analyses is thus relieved to some extent and the work of analysis is greatly simplified because more comprehensive lesser problems are made available as units.

This pyramiding of solutions has already been demonstrated by the methods used in solving the stock of problems treated in the preceding chapters. Further work in graphic description is an inherent part of a study of graphic computation, hence an opportunity not only to apply these stock solutions but to increase their number will be provided.

88. Nature of Graphic Computation. — A computation is a transformation of information from the form in which it originally exists to the form in which it is desired. The information dealt with in this type of *graphic computation* is concerned with the in-space relations of points, lines, surfaces and solids. The specific information sought by graphic computation is in the form of linear and angular dimensions.

Graphic description is employed as the means of compiling and transforming existing information so as to make available the true value descriptions of the dimensions sought. The result of a graphic computation is therefore expressed by a true length representation of a line which defines a desired linear dimension or by a true value representation of the angle between two lines which define a desired angle.

To execute a graphic computation the draftsman must first investigate both the requirements of the solution and the available information. Next he must mentally recognize the lines in space which define the required dimensions and then devise a scheme for establishing and describing these lines. Lastly he must augment these descriptions as found necessary in order to make available true representations of the dimensions sought.

It is the object of this chapter to supply the student with a working stock of solutions for fundamental problems in graphic computation.

89. To Determine the Distance Between Any Two Given Points. — Let it be required to determine the distance between the two points described in Fig. 335*a*.

Analysis: The two given points determine a straight line, the length of which is the required dimension.

Procedure of Solution: (*a*) Add the description of the line to the drawing, see Fig. 335*b*.

(*b*) Describe this line after it has been rotated until parallel to a reference plane, using the cone method explained in Art. 48, see Fig. 335*c*. The representation upon the reference plane to which the line is parallel shows the

true length of the line, hence the true distance between the given points.

An alternative method is shown in Fig. 335*d*. The true distance between the points is shown in this case by the representation of the line on a new reference plane established parallel to the line.

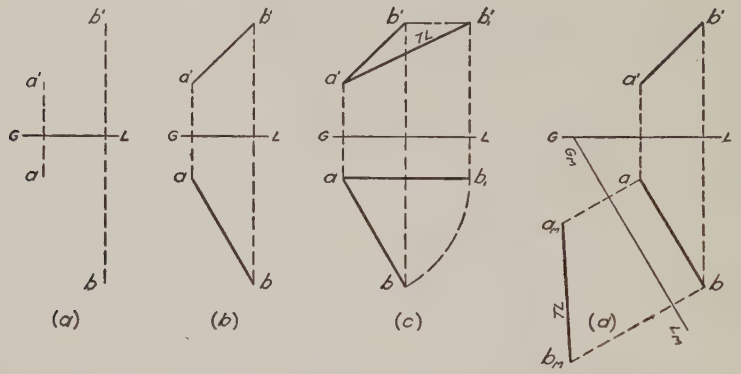


Fig. 335. To Determine the Distance between Any Two Given Points.

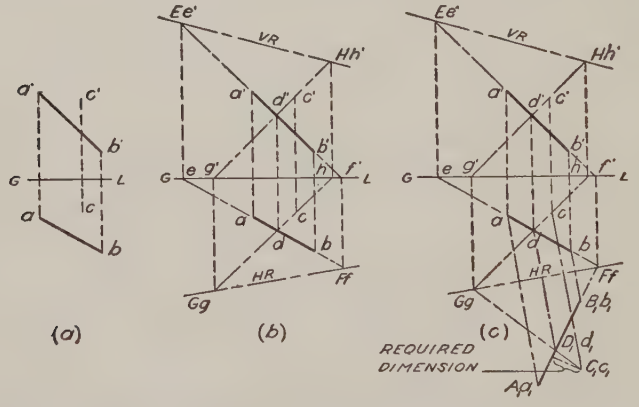


Fig. 336. To Determine the Shortest Distance between a Given Point and a Given Line.

90. To Determine the Perpendicular Distance Between a Given Point and a Given Straight Line. — Let it be required to determine the perpendicular distance between the point and the line described in Fig. 336*a*.

Analysis: The direction of the required dimension is along a line which passes through the given point and which intersects the given line at right angles. The length of this line between the given point and the given line is the required dimension.

The defining line may be established by the following method. First pass a plane through the given point and the given line. Next rotate the point and line about a trace of this plane into a reference plane. The true relation of the point and line are shown in this rotated position, hence the true length representation of the required defining line may be added.

Procedure of Solution: (*a*) Any convenient line is passed through the given point to intersect the given line. The plane determined by this line and the given line contains the given line and the given point, see Fig. 336*a*.

(*b*) This plane is now rotated about its H-trace into the H-plane. Since this operation is performed solely to describe the given point and the given line in their true relationship, only this point and line need be rotated.

A convenient and accurate method of performing this rotation is illustrated in Fig. 336*b*. The H-piercing points, being on the axis of rotation do not change in position during the rotation, hence one point on the representation of each line in its rotated position is already known. If the point of intersection of the two lines is rotated into H, a second point on the representation of each line is established and the representations may be drawn. The representation of the given point in its rotated position must be on the corresponding representation of the line passed through it; hence it may now be located.

(*c*) The representation of the required perpendicular line in this rotated position may now be added to the drawing, see Fig. 336*c*. This representation shows the true length of the line, hence the true value of the required dimension. If a complete graphic description of this line is required it may be obtained by counter-rotating.

91. To Determine the Perpendicular Distance from a Given Point to a Given Plane. — Let it be required to determine the perpendicular distance between the point and plane described in Fig. 337*a*.

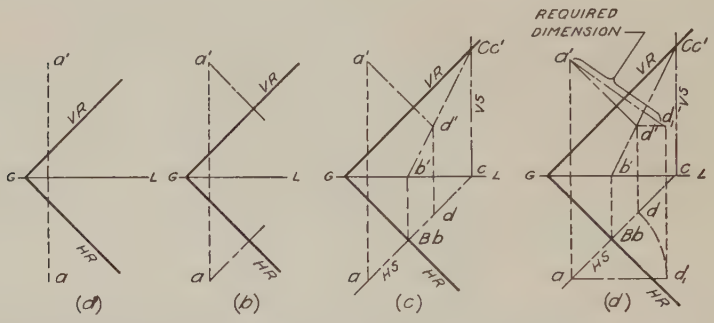


FIG. 337. To Determine the Shortest Distance from a Given Point to a Given Plane.

Analysis: The required dimension is the length of a perpendicular line from the given point to the given plane, the length being measured between the given point and the point at which the line intersects the plane.

Procedure of Solution: (*a*) Establish a line through the given point and perpendicular to the given plane, see Fig. 337*b*.

(*b*) Determine the point at which this new line intersects the given plane, see Fig. 337*c*.

(*c*) Show a true length representation of the line. The distance between the representation of the given point and the representation of the point at which the line intersects the plane is the required dimension, see Fig. 337*d*.

92. To Determine the Vertical Distance from a Given Point to a Given Plane. — Let it be required to determine the distance in a vertical direction from the point to the plane described in Fig. 338*a*.

Analysis: The direction of the required dimension is along a line through the given point and perpendicular to the H-plane. The distance between the given point and

the point at which this line intersects the given plane is the required dimension.

Procedure of Solution: (*a*) Establish a line through the given point and perpendicular to the H-plane, see Fig. 338*b*.

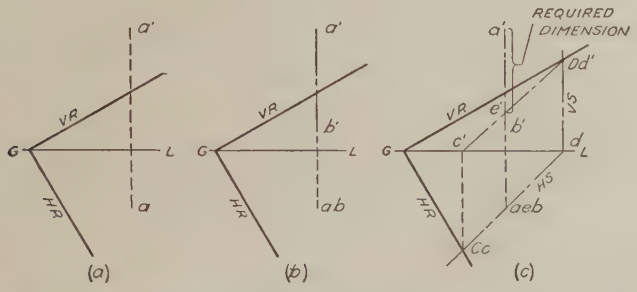


FIG. 338. To Determine the Vertical Distance from a Given Point to a Given Plane.

(*b*) Determine the point at which this line intersects the given plane, see Fig. 338*c*. The defining line of the dimension is parallel to V, hence the true dimensions is shown between the V-representation of the given point and the V-representation of the point at which the line intersects the given plane.

93. To Determine the Distance Between a Pair of Skew Lines. — Let it be required to determine the distance between the two non-intersecting and non-parallel lines described in Fig. 339*a*.

Analysis: The shortest distance between such a pair of lines is equal to the distance from any point on one of the lines to a plane which is parallel to that line and contains the other.

Procedure of Solution: (*a*) First, describe a line which is parallel to one of the given lines and which intersects the other; then describe a plane which contains these two intersecting lines, see Fig. 339*b*.

(*b*) Next, from any convenient point on the line outside the plane, establish a line perpendicular to the plane.

Determine and describe the point at which this line intersects the plane, see Fig. 339*c*.

(*c*) Finally, show a true length representation of this perpendicular line, see Fig. 339*d*. The distance between the representation of the point at which the line intersects the plane and the representation of the point on the skew line is equal to the shortest distance between the skew lines.

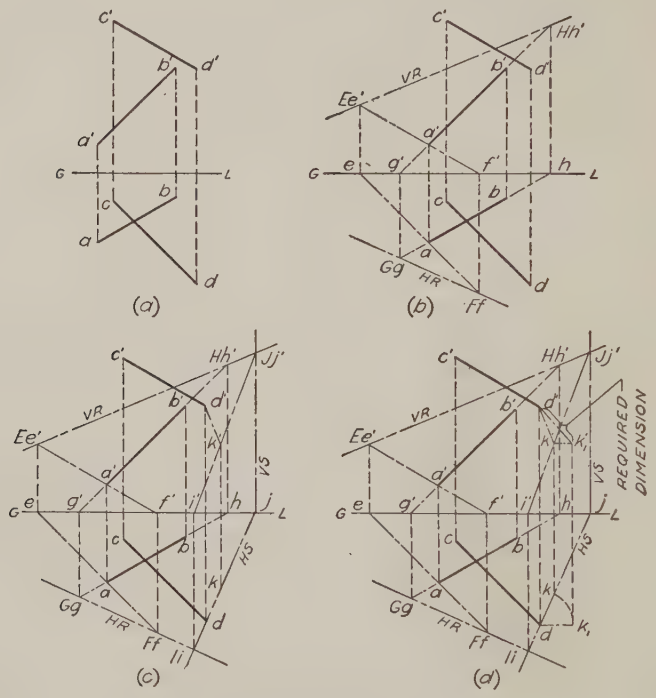


Fig. 339. A Method of Determining the Shortest Distance between a Pair of Skew Lines.

94. To Determine the Distance Between Two Parallel Planes. — Let it be required to determine the distance between the two parallel planes described in Fig. 340*a*.

Analysis: The distance between the two planes is equal to the perpendicular distance from a point in one plane to the other plane. Hence this problem is essentially the same

as that of determining the perpendicular distance from a point to a plane.

Procedure of Solution: (*a*) Describe a line which lies in one of the planes and select a convenient point on this line from which to establish a line perpendicular to both planes, see Fig. 340*b*.

(*b*) Determine the point at which this perpendicular line pierces the second plane and show a true length representation of the line, see Fig. 340*c*. The distance between the representations of the points of this line which lie in the respective planes is the distance between the planes.

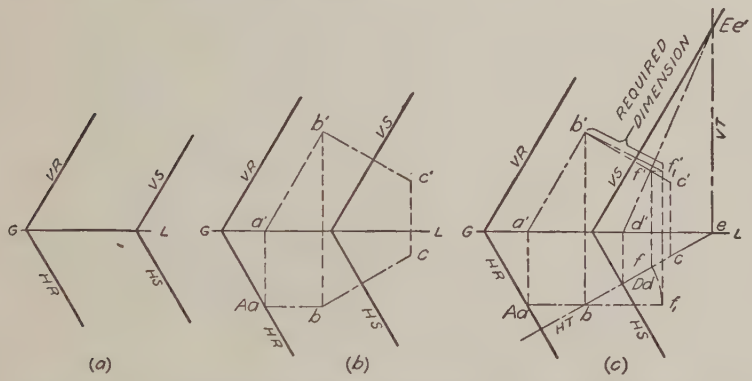


Fig. 340. To Determine the Distance between a Pair of Parallel Planes.

95. To Determine the Angle Between a Pair of Intersecting Lines. — Let it be required to describe the true angle between the intersecting lines shown in Fig. 341*a*.

Analysis: The true angle between a pair of intersecting lines is represented on a reference plane when the plane of the intersecting lines is parallel to, or coincident with, that reference plane. A desirable arrangement of the lines relative to a reference plane may be established by any one of three methods:

(1) The lines may be rotated into a reference plane about a trace of a plane which contains them.

(2) The lines may be rotated parallel to a reference plane

about an axis which is parallel to that reference plane and which lies in the plane of the lines.

(3) A new reference plane may be established parallel to the plane of the lines.

Procedure of Solution — Method 1: (*a*) Describe the plane which is determined by the intersecting lines, see Fig. 341*b*.

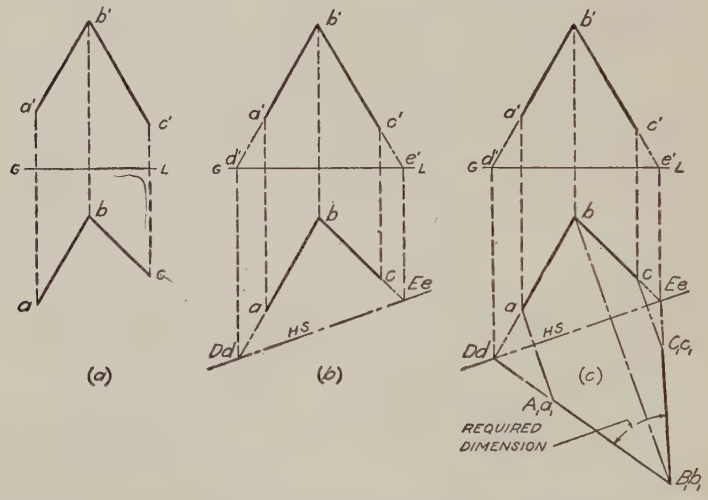


FIG. 341. Determining the True Angle between a Pair of Intersecting Lines by Rotating the Lines into a Reference Plane.

(*b*) Using a trace of this plane as an axis, rotate the intersection of the lines into a reference plane. The piercing points of the lines on this trace do not change their position during rotation, hence they, with the representation of the intersection of the lines in its revolved position, determine the representations of the lines in their revolved positions, see Fig. 341*c*. The angle between the representations of the revolved lines is a true representation of the angle between the lines in space.

Procedure of Solution — Method 2: (*a*) First, establish an axis in the plane of the lines and parallel to a reference plane. This may be accomplished by assuming a line which

is parallel to the desired reference plane and which intersects both of the given lines, see Fig. 342*a*.

(*b*) Next, rotate the point at which the given lines intersect about this axis until the radius of the point is parallel to that reference plane to which the axis is parallel. The points at which the given lines intersect the axis do not change their position during rotation, hence the representations of the lines in their revolved position may be added, see Fig. 342*b*. Since in this revolved position the lines are parallel to the reference plane to which the axis is parallel, the angle between the representations of the lines on this reference plane is the true angle between the lines in space.

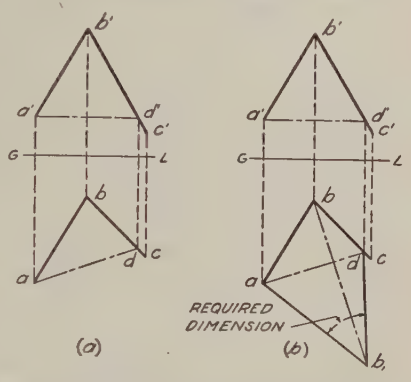


Fig. 342. Determining the True Angle between a Pair of Intersecting Lines by Rotating the Lines Parallel to a Reference Plane.

Procedure of Solution — Method 3: (*a*) First, establish a third line parallel to a primary reference plane and in the plane of the intersecting lines, see Fig. 343*a*.

(*b*) Next, establish a new reference plane perpendicular to this third line and represent the lines on this new reference plane, see Fig. 343*b*.

(*c*) Establish a second additional reference plane using the first as a mate and parallel to both of the intersecting lines. The true angle between the lines is represented on this plane, see Fig. 343*c*.

96. To Determine the Angle Between a Line and a Plane. — The angle between a line and a plane is measured in a second plane which contains the line and is perpendicular to the first plane. The two lines which define the angle are the given line and line of intersection of the two planes, see Fig. 344.

Upon initial consideration it would seem that the problem should be solved by establishing the defining lines of the

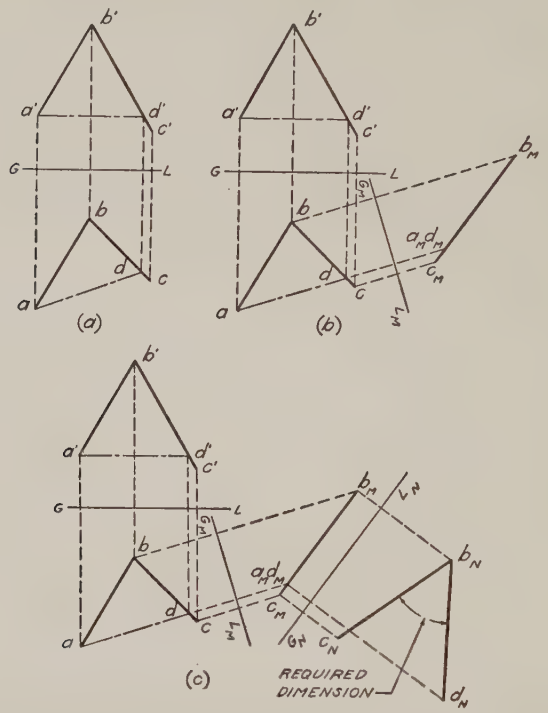


Fig. 343. Determining the True Angle between a Pair of Intersecting Lines by Establishing a Reference Plane Parallel to the Lines.

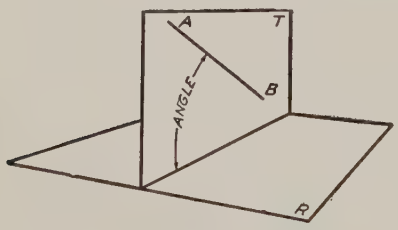


Fig. 344.

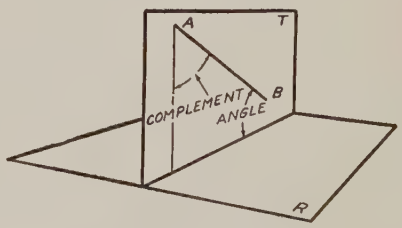


Fig. 345.

angle. Further consideration reveals the fact that any line perpendicular to the given plane and intersecting the given line makes with the given line an angle which is the com-

plement of the required angle, see Fig. 345. Such a line may be so easily established that it is usually expedient to solve for the complement of the desired angle rather than for the angle itself. The following example illustrates this method.

Let it be required to determine the angle between the line and the plane described in Fig. 346*a*.

Analysis: The angle between the given line and a second line which intersects it and is perpendicular to the given plane is the complement of the required angle. If such a second line is assumed and the true angle it makes with the given line is represented a simple geometric construction may be employed for determining the required angle.

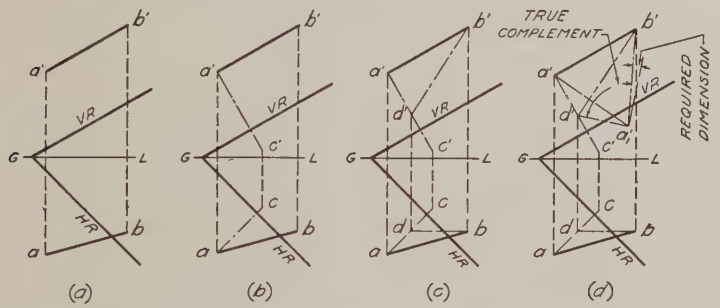


Fig. 346. A Method of Determining the True Value of the Angle between a Given Line and a Given Plane.

Procedure of Solution: (*a*) First, describe an assumed line perpendicular to the given plane and intersecting the given line, see Fig. 346*b*.

(*b*) Next, establish an axis of rotation about which these two intersecting lines may be rotated until parallel to a reference plane, see Fig. 346*c*.

(*c*) Finally, represent the lines after rotating them parallel to a reference plane, see Fig. 346*d*. The representation of the angle on the plane to which the lines are parallel is true size. By constructing a line perpendicular to the representation of one of the lines at their intersection, the true value of the desired angle is made available.

97. To Determine the Angle Between Two Intersecting Planes. — Let it be required to determine the angle between the two intersecting planes described in Fig. 347*a*.

Analysis: A third plane perpendicular to the intersection of the given planes cuts a line from each. The angle between these two lines is the angle between the planes, see Fig. 348.

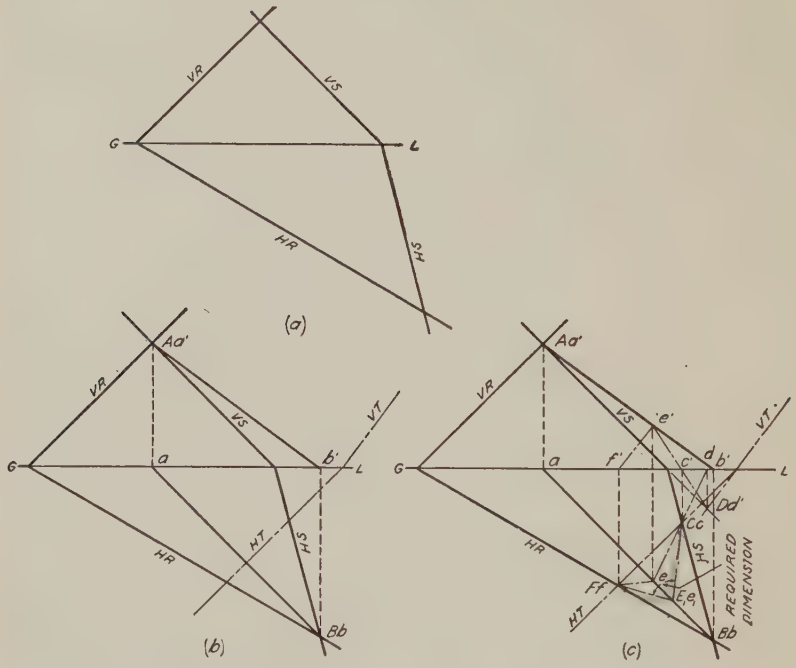


FIG. 347. The Logical but Lengthy Method of Determining the True Angle between a Pair of Intersecting Planes.

Procedure of Solution: (*a*) First, determine the intersection of the two given planes and establish a third plane perpendicular to this line of intersection, see Fig. 347*b*.

(*b*) The two lines, one in each of the given planes, cut by this third plane are the defining lines of the desired angle and may be established by determining the intersections of the third plane with the given planes. By rotating these lines about a trace of the third plane into a reference plane

the true value of the required angle may be shown, see Fig. 347*c*.

The logical procedure outlined above for establishing the defining lines of the required angle may be considerably shortened in many cases due to the wealth of information available concerning these lines. An examination of the in-space relations of the data reveals the fact that a triangular plane figure is formed by the two lines cut from the given planes and a trace of the third plane, see Fig. 349. An inspection of this plane figure with the view of collecting information which would enable the defining lines to be

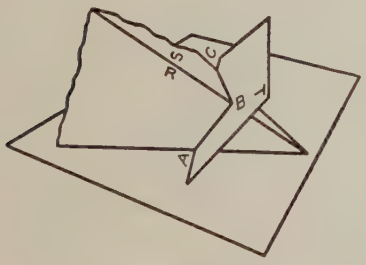


Fig. 348.

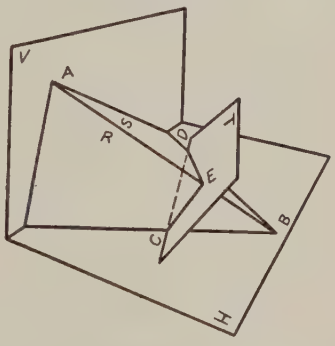


Fig. 349.

established shows that an abundance of such information is available. The directions of the lines are known to be perpendicular to the intersection of the given planes; their common point is known to be the point at which the intersection of the given planes intersects the third plane; their piercing points are known to be the points at which a trace of the third plane intersects the corresponding traces of the given planes; etc. These facts and others make it possible to establish the defining lines by any one of a number of procedures based upon either of the two methods of establishing a line. The procedure outlined below employs the point-and-direction method for determining one of the defining lines and the two-point method for determining the other.

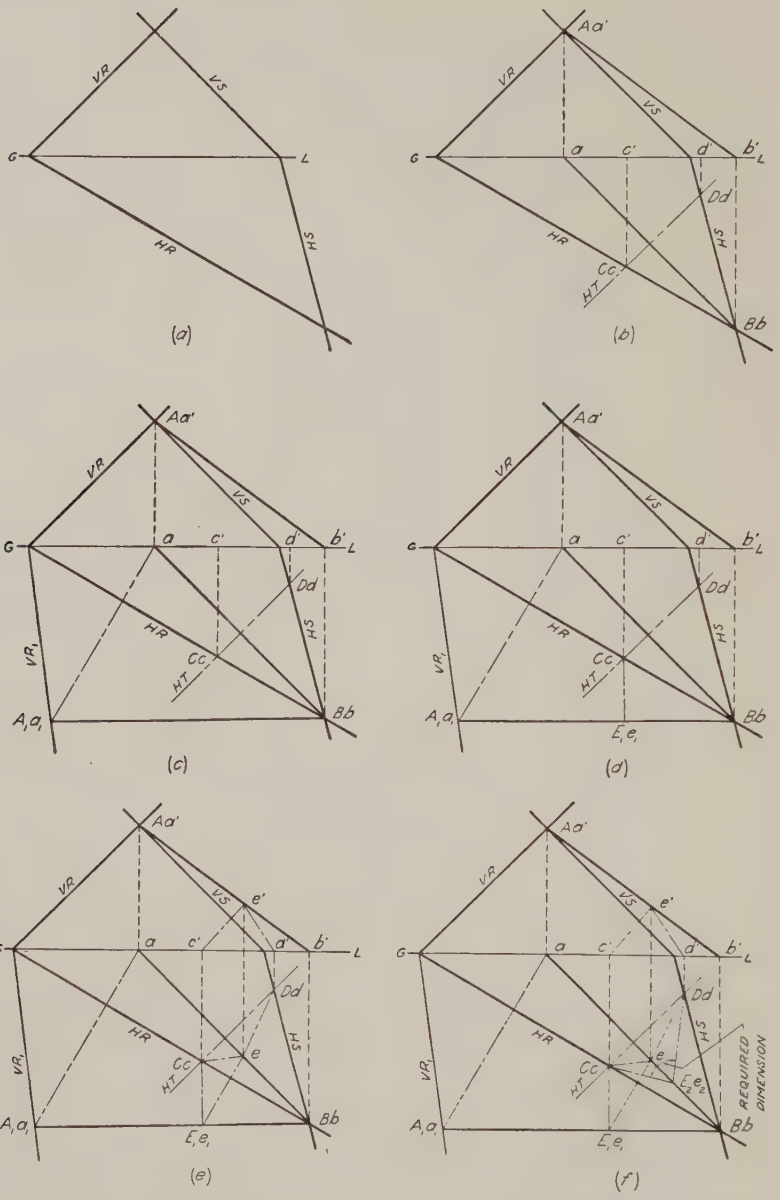


FIG. 350. A Preferred Modification of the Full Logical Method of Determining the True Angle between a Pair of Intersecting Planes.

(*a*) First, establish a point on each of the defining lines by determining the points at which the H-trace of the third plane intersects the H-traces of the given planes, see Fig. 350*b*.

(*b*) If one of the given planes were revolved about a trace into a reference plane, that defining line which is in this plane and the intersection of the two given planes would be shown at right angles in this revolved position. The intersection of the two given planes may be readily represented in this revolved position, see Fig. 350*c*. The representation of the defining line may now be added as the representation of one end is available and its direction is known, see Fig. 350*d*. A counter-revolution establishes this defining line in its normal position and enables the second defining line to be added, see Fig. 350*e*.

(*c*) The final step consists of rotating the defining lines into a reference plane about a trace of the cutting plane, see Fig. 350*f*. The true angle between the lines is thus made available.

CHAPTER XII

DEVELOPMENT OF SURFACES

98. Introduction. — The act of rolling out a surface of higher degree to form a plane surface is called *developing* the surface. The plane figure resulting from this rolling out process is called a *development* of the original surface. The original surface may be reformed by rolling up its development.

To be truly developable a surface must be of such nature that it is unnecessary to stretch or otherwise distort it during the process of rolling it out. Obviously none of the

Fig. 351. The Process of Developing Surfaces.

doubly curved surfaces, such as spheres, can be truly developed. Only those singly curved surfaces can be truly developed which have adjacent elements in the same plane, as only these surfaces can be rolled out in such a manner that their elements come successively in contact with the plane of development, see Fig. 351. The truly developable surfaces include therefore, only the cone, the cylinder, and the convolute. The convolute does not find as wide an application as the cone and cylinder and therefore will not be considered in this brief treatment.

Schemes of approximating the development of warped and doubly curved surfaces and of blanks for drawing and pressing, have been devised to meet practical requirements.

Inasmuch as these schemes are of special nature they also are omitted from this treatment.

99. Method of Developing a Cylindrical Surface. — The elements of a cylindrical surface not only remain parallel during the process of rolling out its development but also remain the same distance apart, see Fig. 351. These facts provide the basis for the conventional method of drawing the development of this surface.

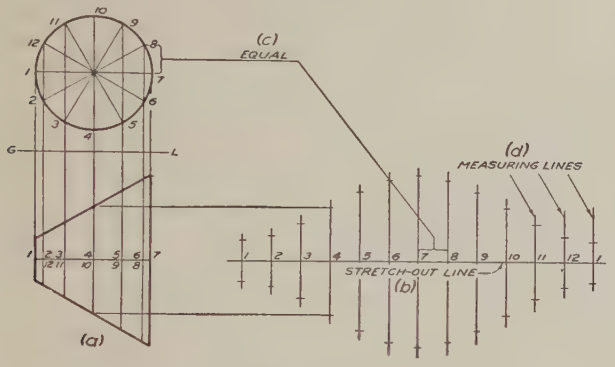


Fig. 352.

As a preparatory step, a number of elements are established on the surface to be developed, see Fig. 352*a*. These elements should be equally spaced for sake of convenience and may be located with the standard triangles.

A *stretch-out line* is the first line of the development to be drawn. This is the straight line along which a selected circumference of the cylinder would roll during the development process, see Fig. 352*b*.

The developed positions of the elements are next located by spacing points along the stretch-out line. The length of the cords between the elements of the original surface are customarily used for spacing these points as this practice is well within the usual range of accuracy if the elements are spaced at 15° to 30° and the drawing is of usual size, see Fig. 352*c*.

Measuring lines, lines along which the true lengths of the elements are to be laid off, are now added at the positions determined by the preceding step. These lines, to agree with the developed positions of the elements, must be perpendicular to the stretch-out line, see Fig. 352*d*.

Confusion may be avoided by notating corresponding elements and measuring lines. This involves two practical considerations. The first is concerned with the fact that in practice it is desirable to avoid long seams when forming sheet metal parts from developments. The second is con-

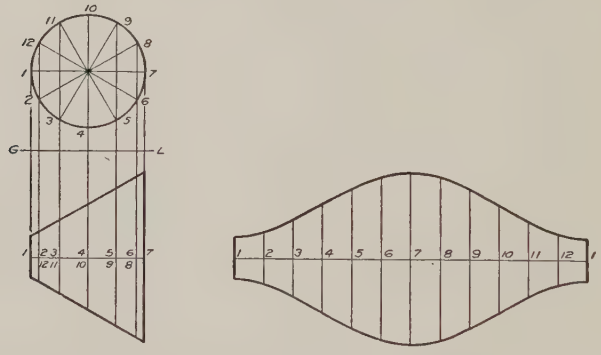


Fig. 353. A Completed Development of a Cylindrical Surface.

cerned with the fact that the outside surface rather than the inside is customarily shown by a development. Because of the first consideration, the surface must be cut along its shortest element and this element located at the extremities of the development. The notation of the developed positions of other elements must be in agreement with the second consideration; that is, the order of locating the elements must be such that the outside surface is shown, see Fig. 352*e*. It should be noted that the illustrations in Fig. 351 are incorrect in this respect.

After establishing and identifying the necessary measuring lines the true lengths of the elements are laid off along them. The stretch-out line in this example is the line of

symmetry of the development, hence it equally divides the developed elements.

An irregular curve is used when completing the drawing, see Fig. 353.

100. Method of Developing a Conical Surface. — An investigation of the process of developing the conical surface, illustrated in Fig. 351, reveals the several facts which supply the basis for the conventional method of drawing this development.

Referring to Fig. 351 it may be seen that the apex of the conical surface remains stationary during the process of rolling out the development and that all elements of the surface intersect at this point when in their developed positions.

It may further be seen that the above mentioned point is the center of the circular stretch-out line along which a circumference of the cone rolls during the process of development. The radius of any circular stretch-out line is equal to the true length of an element of the original surface between the apex and the circumference which corresponds to the stretch-out line.

The length of a circular stretch-out line is equal to the length of the corresponding circumference, and the distance between adjacent elements on the stretch-out line is equal to the distance between these elements on the circumference.

The procedure of drawing the development of a conical surface is illustrated by the following example:

Let it be required to develop the conical surface described in Fig. 354.

The first step is to establish a number of equally spaced elements of the surface, see Fig. 355*a*.

The circular stretch-out line corresponding to the circumference employed when establishing the equally spaced elements, is the first line of the development to be drawn, see Fig. 355*b*.

The positions of the elements along this stretch-out line are determined next. The lengths of the cords between the adjacent

elements on the circumference are used for spacing these positions along the stretch-out line, see Fig. 355*c*.

Measuring lines are next established for each element and these are notated in agreement with the customs of cutting surfaces

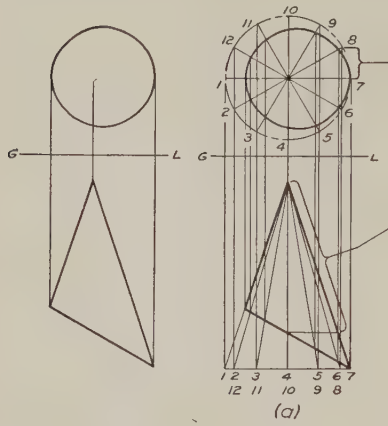


Fig. 354.

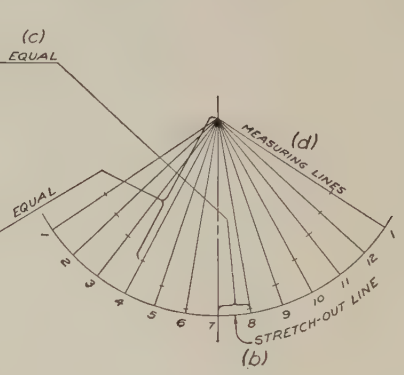


Fig. 355.

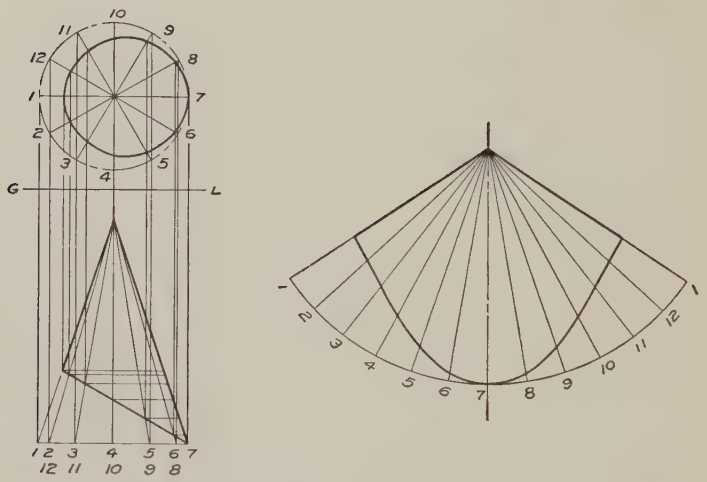


Fig. 356. A Completed Development of a Conical Surface.

at the shortest element and of showing the outside rather than the inside of a surface in its development, see Fig. 355*d*.

The true length of each element is now laid off along its proper measuring line, see Fig. 355*e*, and the development is completed by adding the finished lines, see Fig. 356.

101. To Determine the True Shape of a Non-Orthometrically Positioned Plane Surface. — When developing the entire surface of a solid it is often necessary to determine the true shape and size of a non-orthometrically positioned face. This may be accomplished by rotating the face into, or parallel to, a reference plane or by establishing a new reference plane parallel to the face. Short practical methods are used for applying both these general schemes as illustrated by the following examples:

Example 1. Let it be required to determine the true shape of the oblique base of the prism described in Fig. 357.

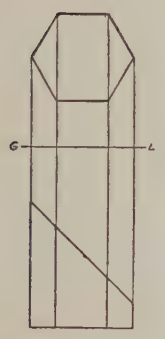


Fig. 357.

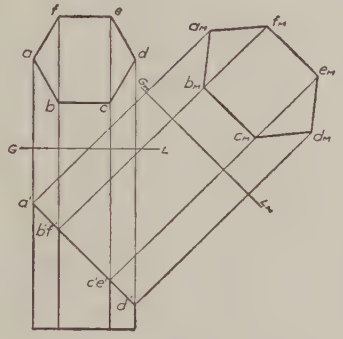


Fig. 358.

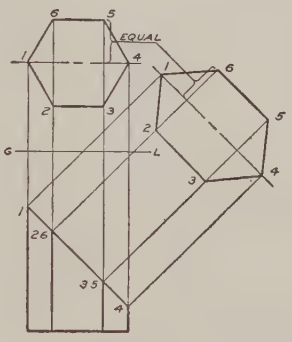


Fig. 359.

The desired true shape may be found by setting up a new reference plane parallel to the oblique base as shown in Fig. 358.

The practical method illustrated in Fig. 359 is obviously a modified application of the new reference plane method.

Example 2. Let it be required to determine the true shape of the oblique base of the cylinder shown in Fig. 360.

The desired true shape may be found by first establishing the plane of the base and by then rotating the base about a trace of this plane into a reference plane, see Fig. 361.

A practical modification of this rotational method is illustrated in Fig. 362. This practical method is advantageous because it permits a choice in the location of the true shape view.

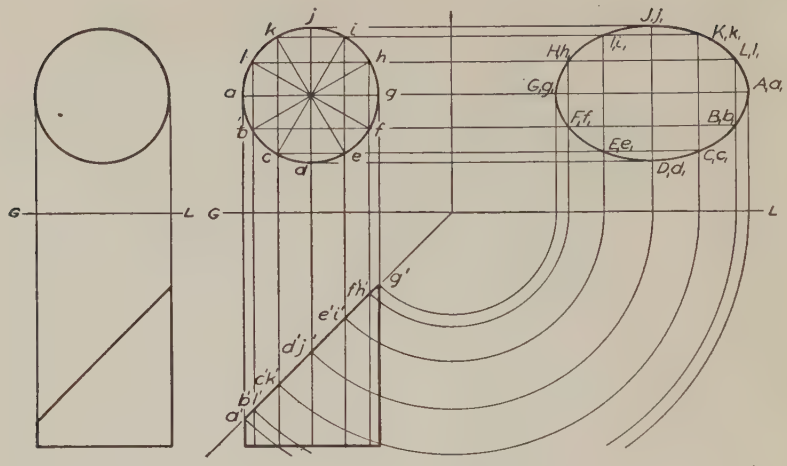


Fig. 360. Fig. 361.

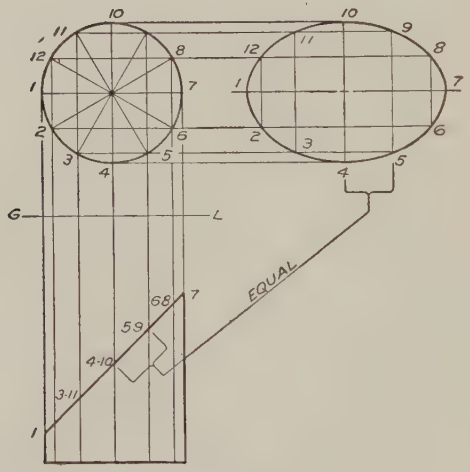


Fig. 362.

102. Development of Combined Surfaces. — If all the individual surfaces of an object are developable, the entire surface of the object may be reformed by combining the reformed developments of the individual surfaces, see Fig. 363. Plane faces, although they do not require development them-

selves, must be included with the developments of other individual surfaces when providing for the development of the entire surface of an object.

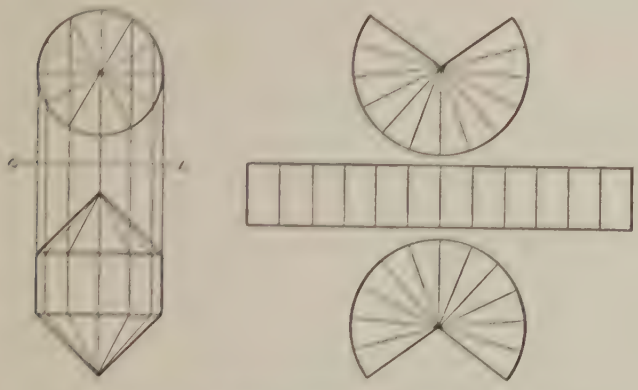


FIG. 363. The Entire Surface of a Solid Developed.

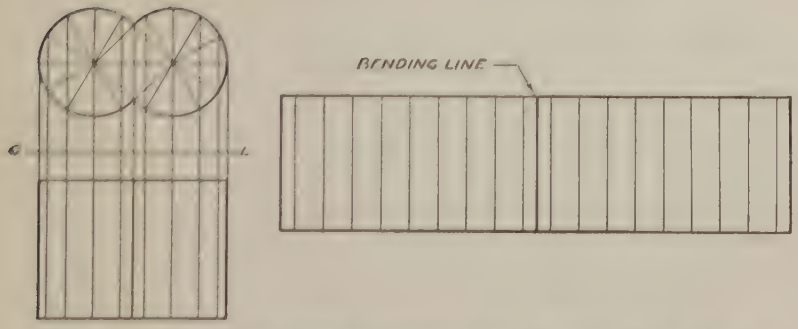


FIG. 364. When Two Surfaces Intersect along a Straight Line their Developments May Be Joined at a "Bending Line."

If two surfaces intersect along a straight line their developments may be joined along this line, see Fig. 364. This joining line is called a *bending line* because the development must be bent along it when reforming the original surfaces. Intersecting plane faces are always joined in this manner because the number of pieces in the entire development of an object is thus reduced and fewer seams are

required to form the surface from the development, see Figs. 365 and 366.

If all the surfaces of a solid intersect, each with another, along straight lines, a one-piece development of the entire

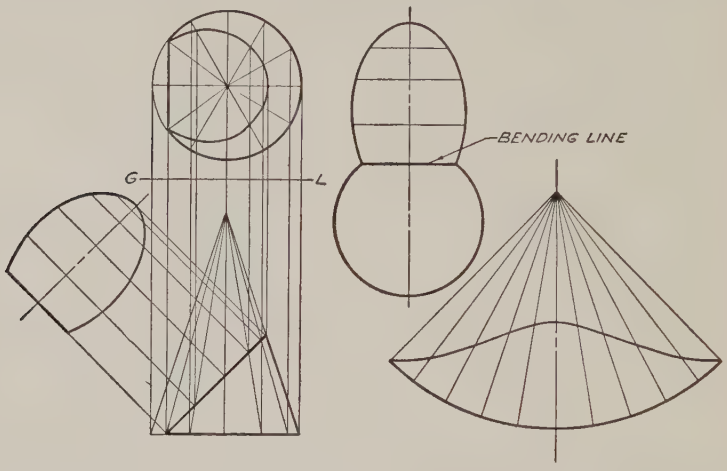


Fig. 365. A Two Piece Development of the Entire Surface of a Solid.

surface may be constructed providing it is possible to avoid overlapping the individual surfaces when in their developed positions. Figs. 367 and 368 illustrate respectively how the entire surfaces of a prism and a pyramid may be thus

Fig. 366.

developed. Stretch-out and measuring lines are employed when developing a series of adjacent plane faces, just as they are employed when developing cylindrical and conical surfaces. Fig. 369 illustrates the use of measuring lines for determining a more complicated development of this nature.

The developed shapes of holes through surfaces are determined by plotting the developed positions of points on the defining lines of the holes. For example, let it be re-

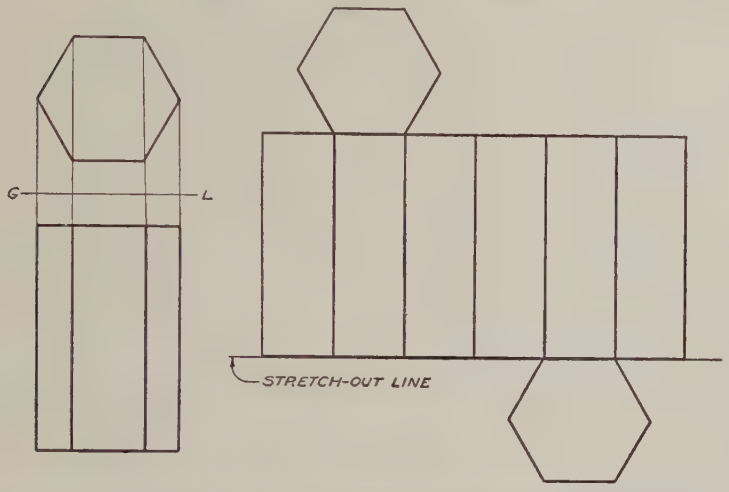


Fig. 367. A One Piece Development of the Entire Surface of a Prism.

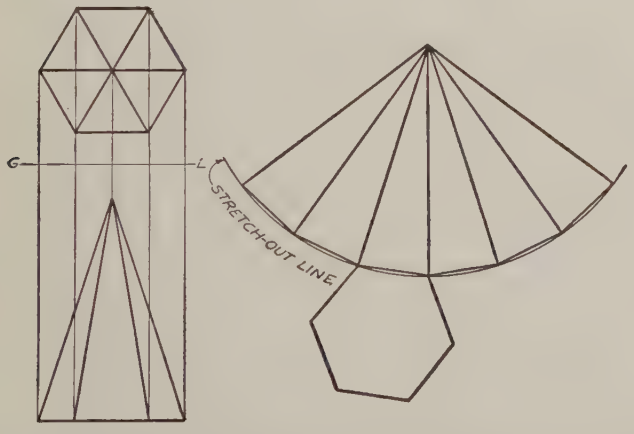


Fig. 368. A One Piece Development of the Entire Surface of a Pyramid.

quired to develop the surface of the vertical prism shown in Fig. 370*a*. The development is first drawn without considering the hole in the lateral surface of the prism caused

by its intersection with the smaller oblique prism. The developed shape of the hole is then determined by plotting the developed positions of the critical points of its defining

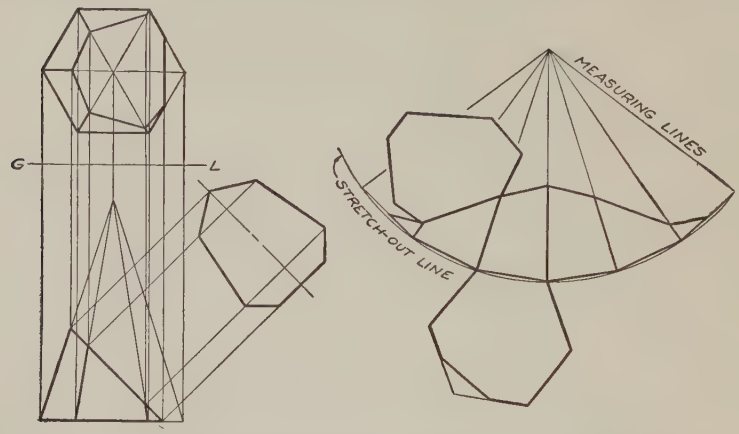


FIG. 369. A Development of the Entire Surface of a Truncated Pyramid.

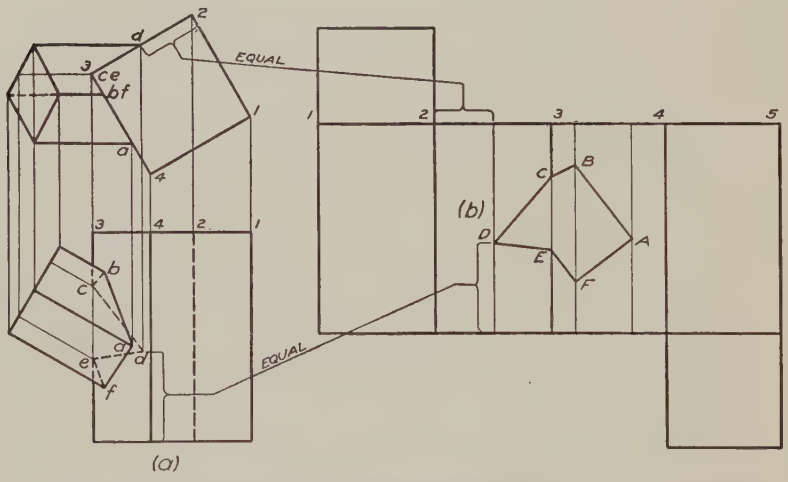


FIG. 370. The Method of Developing a Hole in the Surface of a Solid.

line. Each point is plotted by first establishing the measuring line which passes through it and by then laying off the distance along this measuring line from the stretch-out line to the point, see Fig. 370*b*.

The developed positions of points on the defining line of a hole through a cylindrical or conical surface are plotted in the same manner as explained above. However, the draftsman in this case is confronted with the task of selecting the points which may be employed to best advantage in determining the developed shape of the hole. If the hole is created by an intersection with a second surface, the intersection must be determined before the development can be

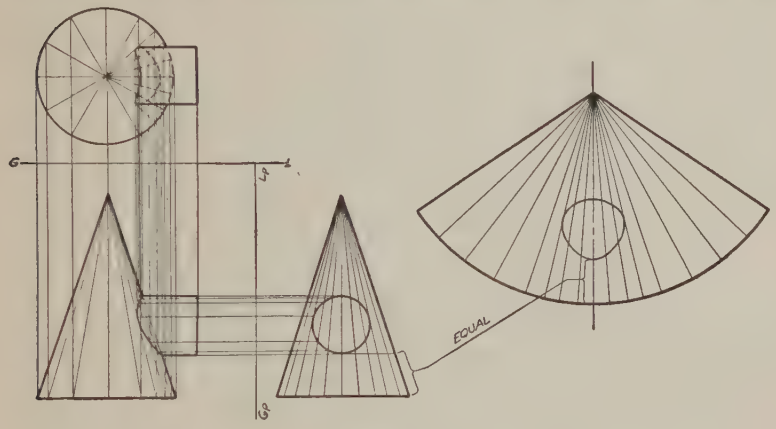


Fig. 371.

started. If, in such a case, the development is of first importance and the description of the intersection is incidental, the points used for establishing the line of intersection should be those which are best suited for determining the developed shape of the hole. Fig. 371 illustrates a drawing executed with this end in view. Note that equally spaced elements were established to assist with the general development of the conical surface and that these are supplemented by others which enabled particularly desirable points to be located on the development of the hole.

RECITATION QUESTIONS AND EXERCISES

The numbers assigned to the following questions and exercises, excepting those for Chapter I, are composed of a whole and a decimal part. The whole number identifies the question or exercise with the text article upon which it is based. The decimal distinguishes the question or exercise from others which refer to the same text article. Thus the first question based upon Art. 50 is assigned the identifying number 50.1.

In many of the exercises definite dimensions are specified for locating the descriptions of points, lines, and planes. When these exercises are worked freehand the specified dimensions should be followed only in respect to proportion and the drawings should be made large for clearness.

QUESTIONS ON CHAPTER I

1. (*a*) Explain in a general manner the work performed during the process of designing a machine.

(*b*) What application does graphic representation find in this work?

2. (*a*) Explain in a general manner the work performed during the processes of constructing a machine.

(*b*) What application does graphic representation find in this work?

3. Compare the functions of graphic representation as applied to design and to construction.

4. In what way is descriptive geometry related to graphic representation?

5. Is knowledge of descriptive geometry of value when making working drawings? Explain.

6. Why is an intimate knowledge of descriptive geometry essential to design drawing?

7. What benefits other than those relating directly to mechanical drawing may be derived from studying descriptive geometry?

QUESTIONS ON CHAPTER II

1.1. Explain why the science of descriptive geometry is based upon a method of graphically describing a point.

2.1. Why is the *location* of a point the only factor to be considered when planning its graphic description?

2.2. (*a*) Why must reference planes be established when devising a scheme for describing the locations of points in space?

(*b*) How are these reference planes arranged relative to each other? Why?

2.3. (*a*) Describe in detail, with the assistance of rough sketches, how the position of a lamp suspended in a rectangular room may be completely specified by three dimensions.

(*b*) Could the lamp's position be specified if less than three such dimensions were given? Explain.

2.4. (*a*) Describe in detail, with the assistance of rough sketches how the position of a lamp suspended in a rectangular room may be *graphically* specified by using the surfaces of the room as reference planes.

(*b*) How many dimensions are required to fix the position of a point in space?

(*c*) Explain how these dimensions are graphically expressed by the method used in answering part (*a*) of this question.

2.5. (*a*) What information is expressed by the graphic record borne by a single reference plane when a set of reference planes are used to graphically record the position of a point in space?

(*b*) Show that the combined graphic records on any two adjacent and perpendicular reference planes are sufficient to express all three of the dimensions necessary to fix the position of a point in space.

3.1. (*a*) What is meant by *orthographic projection?*

(*b*) What is meant by the *representation* of a point?

3.2. (*a*) Show by sketch how the representations of a point are orthographically projected upon a set of reference planes.

(*b*) Explain in detail the information recorded by each of these representations.

3.3. (*a*) Show by sketch how a set of reference planes may be folded into a single plane surface corresponding to a sheet of drawing paper.

(*b*) Assuming that these reference planes bear the representations of a point explain how each representation specifies certain information concerning the position of the point represented.

3.4. Why may a representation of a point on a reference plane be considered a definite graphic statement?

(*b*) How many such statements are required to describe completely the position of a point in space?

(*c*) What is meant by a *pair of reference planes?*

3.5. Show with the aid of sketches that the two representations of a point in space borne by a *pair of reference planes* must be shown on a drawing on a line perpendicular to the intersection of the planes.

QUESTIONS ON CHAPTER III

4.1. Explain briefly the basic principles of graphic description from which the science of descriptive geometry has been developed.

5.1. Why is the formulation of the system of applying the basic principles of graphic description founded upon a consideration of a *point* and a *pair of reference planes?*

5.2. (*a*) What is meant by the *primary pair of reference planes?*

(*b*) How are these planes considered to be arranged in space?

(*c*) What names are applied to these planes?

5.3. (*a*) Show by sketch that the primary pair of reference planes divide all space into four equal parts.

(*b*) Explain with reference to the above sketch what portions of the primary planes would be used when describing the position of points located in each of the four dihedral angles formed by the planes.

6.1. Explain with sketch the method of naming the four dihedral angles and give the established name assigned each angle.

7.1. Explain with sketch the established method of projecting representations upon the primary pair of reference planes.

8.1. Explain and illustrate the method of notating representations projected upon the primary reference planes.

9.1. Explain and illustrate the established method of folding the primary pair of reference planes so as to form a single plane surface corresponding to the surface of a drawing.

10.1. Assume a point to be 2 inches in front of V and 3 inches above H. Explain and illustrate the steps in the procedure of executing a graphic description of this point.

10.2. Assume a point to be 2 inches in front of V and 3 inches below H. Explain and illustrate the steps in the procedure of executing a graphic description of this point.

10.3. Assume a point to be 2 inches in back of V and 3 inches above H. Explain and illustrate the steps in the procedure of executing a graphic description of this point.

10.4. Assume a point 2 inches in back of V and 3 inches below H. Explain and illustrate the steps in the procedure of executing a graphic description of this point.

10.5. Justify the following, using a simple example as an illustration: "Every representation of a point is a graphic statement which in conjunction with other such statements specifies the position of the point in space." Tell exactly what information is specified by each representation and how this information can be obtained from a drawing.

10.6. (*a*) What purpose does a *ruled projector* serve in graphic descriptions of points?

(*b*) Why should ruled projectors be equal in visibility with notation?

(*c*) Why should ruled projectors be of greater visibility than ground lines?

(*d*) How is greater visibility given to ruled projectors? Why?

(*e*) What dimensions should be used when making the dashes and spaces of a ruled projector?

11.1. Show by sketch how the application of the established methods of projecting upon the primary reference planes and of folding these planes into the drawing paper, creates four distinct types of graphic descriptions, one for each of the four dihedral angles.

11.2. Explain with the aid of sketches the characteristic features of each of the four types of graphic descriptions.

12.1. Why were the rules which govern the methods of employing all reference planes formulated from a consideration of the single pair of reference planes known as the primary pair of reference planes?

12.2. Has the establishment of the four types of graphic description any

bearing upon the custom of considering reference planes in pairs? Explain fully.

12.3. Explain the requirements which must be met, when adding a new reference plane, in respect to arrangement, method of projecting upon it and method of folding it into the drawing sheet.

12.4. Explain why the dihedral angle in which a new reference plane is added must be considered when deciding the order of projecting upon the new plane.

12.5. Does the choice of the mating plane for an additional reference plane have any effect upon the order of projecting upon the new plane? Explain fully giving examples in each of the four angles of description.

12.6. Explain how additional reference planes must be folded in order to maintain the characteristics of a particular type of graphic description. Include all angles of graphic description and all possible arrangements of additional planes in this explanation.

13.1. (*a*) What is a *profile plane?*

(*b*) Why is there a choice of ground lines about which a profile plane may be folded?

(*c*) Does the choice of ground lines for a profile plane affect the manner of projecting upon it? Explain for all four angles of description.

14.1. (*a*) Explain why there is a confliction of rules when a profile plane is used in the description of several points located in different dihedral angles?

(*b*) Does a confliction of rules always exist if but two angles of description are involved?

(*c*) How is confusion avoided when the confliction of rules discussed above is encountered?

15.1. Explain the system of notation employed for additional reference planes.

QUESTIONS ON CHAPTER IV

16.1. Explain briefly how the function of graphic representation as applied in design drawing differs from the function it serves in working drawing.

16.2. "Descriptive geometry enjoys two widely differing applications." Explain.

16.3. What constitutes a working knowledge of descriptive geometry?

17.1. (*a*) Define a straight line in terms of the unit of description.

(*b*) Show by sketch how the representations of a line in the first angle are obtained and explain the method of notating both the actual line in space and the representations of the line shown on the drawing.

17.2. (*a*) What relation has the representations of a line to the representations of the points which make up the line?

(*b*) Make properly labeled and notated drawings of four lines one in each of the dihedral angles.

18.1. Explain the system of abbreviating the information supplied for the purpose of specifying given points in space.

18.2. Describe the positions of the following points both graphically and verbally, A, $6 + 2 + 1$; B, $8 - 3 + 2$; C, $10 + 3 - 4$; D, $6 - 2 - 3$; F, $8 + 0 + 2$; G, $10 + 2 + 0$; H, $6 - 0 - 0$.

18.3. Verbally explain the positions of the points described in Fig. 39.

19.1. How are the data concerning a given line expressed in the statement of problems?

19.2. Describe the following lines both graphically and verbally, $AB(6+3+2)\ (8+2+3)$; $CD(8-1-3)\ (10-4-2)$; $EF(4+2-3)\ (6+1-4)$; $GH(8-3+2)\ (10-4+3)$.

19.3. Verbally explain the positions of the lines described in Fig. 41.

20.1. (*a*) Define a plane in terms of the descriptive unit.

(*b*) Is the method employed for graphically describing lines suitable to the graphic description of planes in space? Explain.

20.2. Explain the established method of graphically describing planes in space.

20.3. (*a*) What is meant by the *trace of a plane?*

(*b*) Define *horizontal trace.*

(*c*) Define *vertical trace.*

(*d*) What is meant by the *primary traces of a plane*?

20.4. Explain the method of notating planes and the traces of planes.

20.5. (*a*) Explain why *representations* of traces are not employed in the graphic descriptions of planes.

(*b*) Make a drawing showing a properly notated graphic description of an assumed plane in space.

(*c*) Explain with reference to the above drawing where the representations of the traces would be located if they were shown on the drawing.

21.1. Explain the general scheme employed for specifying information concerning given planes.

21.2. Explain the abbreviated method of supplying information concerning given planes when their traces intersect.

21.3. (*a*) When do lines of the same plane intersect?

(*b*) Will the traces of a plane always intersect? Explain.

21.4. (*a*) Make drawings of the following planes:

R 1, 3, R 45°, L 30°;	S 3, 6, R 30°, L 30°.
T 2, 8, L 45°, R 60°;	V 4, 10, L 30°, L 60°.
W 1, 2, 90°, R 45°;	X 3, 4, R 45°, 90°.

(*b*) State the angle in which each of the above planes is given and also state the position of each relative to the primary reference planes.

21.5. Make drawings of the following planes:

(*a*) R in the first angle, traces meet at (6), HR 30° R, VR 60° R.

(*b*) S in second angle, traces parallel to G. L., HR 2 inches from G. L., VR 3 inches from G. L.

21.6. Verbally explain the characteristics of the planes described in Figs. 47 and 48.

22.1. Explain how solids are graphically described.

22.2. The characteristic lines of prisms and pyramids are represented and notated on every reference plane employed. Is this true for cylinders and cones? Explain.

22.3. How are the representations of solids which have surfaces of revolution, represented and notated? Illustrate.

23.1. Graphically describe the following solid: $AB(6 + \frac{1}{2} + \frac{1}{2})$ $(8 + \frac{1}{2} + \frac{1}{2})$ is one edge of the base of a right square pyramid having an altitude of 3 inches. The base of the pyramid is wholly in first angle and is parallel to H.

23.2. Graphically describe the following solid: $AB(6 - \frac{1}{2} - 2)$ $(6 - 3\frac{1}{2} - 2)$ is the axis of a right circular cone. A is the apex. The base is parallel to H.

24.1. (*a*) Explain with the aid of sketches how an additional reference plane is used in the first angle. Cover fully the order of projecting upon the new plane and the manner of folding it into the drawing surface. Confine the discussion first to a case in which the H-plane is used as a mate to the new plane and then repeat for a case when the V-plane is used as a mate.

(*b*) Upon what fundamental principles of graphic description are the above procedures based?

24.2. Explain with the aid of sketches how an additional reference plane is used in the second angle. Follow the outline given in 24.1.

24.3. Explain with the aid of sketches how an additional reference plane is used in the third angle. Follow the outline given in 24.1.

24.4. Explain with the aid of sketches how an additional reference plane is used in the fourth angle. Follow the outline given in 24.1.

24.5. (*a*) Graphically describe an assumed point in the first angle.

(*b*) Add to the above drawing the representation of the point on an assumed additional reference plane which is used with H as a mate.

(*c*) Add another representation of the point upon a second assumed additional reference plane using V as its mate.

(*d*) Add still another representation of the point upon a third additional plane which uses the first added reference plane as a mate.

24.6. Repeat Exercise 24.5 but assume the point in the second angle.

24.7. Repeat Exercise 24.5 but assume the point in the third angle.

24.8. Repeat Exercise 24.5 but assume the point in the fourth angle.

24.9. (*a*) Graphically describe an assumed oblique line in the first angle.

(*b*) Add to the above drawing the representation of this line on an additional plane which is oblique to the line and which uses H as its mate.

(*c*) Add another representation of the line upon a second additional reference plane which is parallel to the line and which uses V as its mate.

(*d*) Add still another representation of the line upon a third additional reference plane which is parallel to the line and which uses the first added reference plane as its mate.

24.10. Repeat Exercise 24.9 but assume the line in the second angle.

24.11. Repeat Exercise 24.9 but assume the line in the third angle.

24.12. Repeat Exercise 24.9 but assume the line in the fourth angle.

24.13. (*a*) Graphically describe an assumed oblique line which has one end in the first angle and the other in the third angle.

(*b*) Add the representation of the line on an additional reference plane which is oblique to the line and which uses H as a mate.

24.14. (*a*) Graphically describe an assumed oblique line having one end in the first angle and the other in the third angle.

(*b*) Add the representation of this line on an additional reference plane parallel to the line and which uses V as its mate.

NOTE: *In the following five exercises the instructor will assign the angle of description to be employed.*

24.15. (*a*) Graphically describe an assumed oblique plane in space.

(*b*) Show the trace of this plane on an additional reference plane which uses the H-plane as its mate.

(*c*) Using a duplicate of the drawing made for part (*a*) of this exercise, show the trace of the plane in space on an additional reference plane which uses the V-plane as its mate.

24.16. (*a*) Graphically describe an assumed plane in space which is perpendicular to H.

(*b*) Show the trace of this plane on an additional reference plane which uses the H-plane as its mate.

(*c*) Using a duplicate of the drawing made for part (*a*) of this exercise, show the trace of the plane in space upon an additional reference plane which uses V as its mate.

24.17. (*a*) Graphically describe an assumed plane in space which is perpendicular to V.

(*b*) Show the trace of this plane on an additional reference plane which uses the H-plane as its mate.

(*c*) Using a duplicate of the drawing made for part (*a*) of this exercise, show the trace of the plane in space on an additional reference plane which uses V as its mate.

24.18. (*a*) Graphically describe an assumed plane in space which is parallel to the G. L.

(*b*) Show the trace of this plane on an additional reference plane which uses H as its mate.

(*c*) Using a duplicate of the drawing made for part (*a*) of this exercise, show the trace of the plane in space on an additional reference plane which uses V as its mate.

24.19. (*a*) Make a drawing showing the principal traces of an assumed plane in space which passes through the principal G. L.

(*b*) Show the trace of this plane on an additional reference plane which uses H as its mate.

(*c*) Why does the task of executing part (*b*) of this exercise differ from that of executing similar parts of the preceding exercises?

24.20. (*a*) Graphically describe an assumed right rectangular prism in the third angle. For sake of convenience take the bases parallel to H.

(*b*) Add the representation of this solid on an assumed additional reference plane which uses H as its mate.

(*c*) Add the representation of the solid on a second assumed additional reference plane which uses V as its mate.

24.21. Repeat Exercise 24.20 but assume the prism in the first angle.

24.22. Repeat Exercise 24.20 but describe a right pentagonal pyramid in the third angle instead of a prism.

24.23. Repeat Exercise 24.20 but describe a right pentagonal pyramid in the first angle instead of a prism in the third angle.

QUESTIONS ON CHAPTER V

25.1. (*a*) Give a definition of a *point* in terms which are suited to the requirements of graphic description.

(*b*) What scheme is suggested by this definition for establishing required points in space?

25.2. Demonstrate with sketches and explain the fact that if a point is on a line its representations must be on the corresponding representations of the line.

25.3. Demonstrate with sketches and explain that if a point is on each of two lines its representations must be on the corresponding representations of both lines.

25.4. (*a*) Make two drawings, one describing a pair of intersecting lines and the other a pair of non-intersecting lines.

(*b*) Explain how these drawings describe the relations of the lines.

25.5. (*a*) Make a drawing describing a pair of non-intersecting lines such that the corresponding representations of the line intersect.

(*b*) Explain how the drawing shows that the lines do not intersect.

(*c*) What information is expressed by the fact that the V-representations of the lines intersect?

(*d*) What information is expressed by the fact that the H-representations of the lines intersect?

25.6. Why is it of utmost importance that a student of descriptive geometry should fully understand and appreciate the significance of the method of establishing and describing required points in space?

26.1. (*a*) What is meant by the *piercing point of a line?*

(*b*) How many piercing points may a line have?

26.2. (*a*) What is meant by the *primary piercing points of a line?*

(*b*) Give the names and definitions of the primary piercing points of a line.

27.1. Why is the method of attacking the problem of determining the description of a piercing point given special attention in the text?

27.2. (*a*) Graphically describe an assumed oblique line in first angle.

(*b*) Let it be required to describe the H-piercing point of this line. Demonstrate the full process of devising a scheme whereby this may be accomplished.

(*c*) State concisely the formal analysis of the problem as deduced by the above process of reasoning.

(*d*) Execute the work of graphically describing the required point.

27.3. Repeat Exercise 27.2 but let the vertical piercing point of the assumed line be the required point.

27.4. Repeat Exercise 27.2 but assume the line in the third angle.

27.5. Repeat Exercise 27.2 but assume the line in the third angle and consider a description of its vertical piercing point to be required.

27.6. (*a*) Graphically describe an assumed oblique line in the first angle.

(*b*) Add the representations of the primary piercing points of this line to the drawing.

(*c*) Assume an additional reference plane which uses the H-plane as its mate and demonstrate the full process of devising a scheme whereby the description of the piercing point of the assumed line with this additional plane may be accomplished.

(*d*) State concisely the formal analysis of the problem of describing the additional piercing point.

(*e*) Execute the work of graphically describing the required point.

27.7. Repeat Exercise 27.6 but let the additional plane be used with V to form a pair of reference planes.

27.8. Repeat Exercise 27.6 but assume the line in the third angle.

27.9. Repeat Exercise 27.6 but assume the line in the third angle and let the additional plane be used with V to form a pair of reference planes.

29.1. Explain the two methods that are available for establishing required straight lines.

29.2. What is the procedure followed in establishing the points which are involved in the problem of establishing required straight lines?

29.3. (*a*) When *direction* is used as a factor in establishing required straight lines, how is it usually fixed?

(*b*) Why does this method of fixing direction find wide usage?

29.4. (*a*) Demonstrate that if two lines in space are parallel their corresponding representations must be parallel.

(*b*) Make drawings of four pairs of parallel lines, one pair in each of the four dihedral angles.

30.1. (*a*) How are planes graphically described?

(*b*) Make drawings describing planes in each of the four dihedral angles.

(*c*) How does this method of description differ from that employed for points and lines?

30.2. "Two points or one point and a direction determine a line in space." Explain on the same basis the factors which are necessary to determine a plane in space.

30.3. "Since planes are described by their traces any method of establishing a required plane in space must lend itself to the establishment of the plane's traces."

Show that two intersecting or parallel lines determine a plane in space and that such lines may be used to advantage in establishing the points which fix the traces of the plane.

31.1. (*a*) Graphically describe three assumed points in the first angle.

(*b*) State concisely the formal analysis of the problem of describing the plane determined by these three points.

(*c*) Add the graphic description of this plane to the drawing.

31.2. Repeat Exercise 31.1 but assume the points in the third angle.

31.3. Repeat Exercise 31.1 but assume one point in the first angle, one in the second and one in the third, and describe that portion of the plane they determine which is in the fourth angle.

31.4. (*a*) Graphically describe an assumed oblique line in the first angle and also an assumed point in the third angle.

(*b*) State the formal analysis of the problem of describing the plane determined by this point and line.

(*c*) Add the description of this plane to the drawing.

31.5. (*a*) Graphically describe two non-intersecting lines in the first angle.

(*b*) Is it possible that a plane in space could contain one of these lines and be parallel to the other?

(*c*) What factors are usually employed to determine a plane in space?

(*d*) Explain how this method of determining a plane in space may be applied to the problem of establishing a plane which would contain one of the lines in the above drawing and be parallel to the other.

(*e*) Add the description of this plane to the drawing.

QUESTIONS ON CHAPTER VI

32.1. (*a*) What kinds of dimensions are employed to fix size?

(*b*) Explain how the above dimensions are measured.

32.2. (*a*) When an object is graphically described for purely descriptive purposes, what lines are represented?

(*b*) What lines in addition to the above are represented on graphic computations?

32.3. Make a fully notated sketch illustrating the graphic computation of a diagonal of an assumed right rectangular prism.

34.1. (*a*) State the analysis of the problem of establishing and describing a line in a plane.

(*b*) Graphically describe the following planes and also establish and describe two lines in each, one by the two-point method and one by the point-and-direction method: Q 1, R 30°, R 45°; R 2, L 60°, R 30°; S 3, R 45°, L 30°; T 4, L 30°, L 60°.

34.2. (*a*) State the analysis of the problem of establishing and describing a line in a plane.

(*b*) Graphically describe the following planes and also establish and describe a line in each: Q 1, 90°, L 30°; R 1, R 30°, 90°; S 1, 90°, 90°; T 1, traces parallel to G. L.; U 1, parallel to H; V 1, parallel to V.

34.3. (*a*) Graphically describe an assumed oblique line in the first angle.

(*b*) Add to the above drawing the descriptions of several planes which contain this line.

34.4. (*a*) What particular points of a plane are directly described by the graphic description of the plane?

(*b*) Explain how other points in the plane may be established and graphically described.

34.5. (*a*) Graphically describe the plane Q 1, R 45°, R 60°.

(*b*) State the analysis of the problem of establishing and describing a point which is in this plane but not on its traces.

(*c*) Add the description of such a point to the drawing.

34.6. Repeat Exercise 34.2 but assume the plane Q to be in the third angle.

34.7. (*a*) Graphically describe an assumed plane R which is in the first angle parallel to the G. L.

(*b*) State the analysis of the problem of establishing and describing a point which is in this plane but not on its traces.

(*c*) Add the description of such a point to the drawing.

34.8. Repeat Exercise 34.7 but assume the plane R to be in the third angle.

34.9. (*a*) Graphically describe an assumed oblique plane S in the first angle and also an assumed point A in the first angle.

(*b*) Is the point A in the plane S? Show and explain the graphic answer to this question.

34.10. Repeat Exercise 34.9 but assume the point A to be in the third angle.

34.11. (*a*) Graphically describe an assumed oblique plane N in the first angle.

(*b*) Add to the above drawing the descriptions of two points A and B which are in the plane N but not on its traces.

(*c*) Add also the description of a line joining the points A and B.

(*d*) Is the line AB in the plane N? State fully the reasons for the answer given.

34.12. (*a*) Graphically describe an assumed plane T which is in the first angle and is perpendicular to H and oblique to V.

(*b*) Add to this drawing the description of several lines and points which lie in the plane T.

(*c*) Would a plane of this nature be more convenient to use than an oblique plane if it were necessary to establish a plane to contain a given point or a given line? Explain fully with illustrations.

34.13. Execute Exercises 34.1, 34.2, 34.5, 34.6, 34.7, or 34.8 as designated by the instructor and include an additional reference plane.

35.1. (*a*) Under what conditions may a point be said to be in a surface?

(*b*) Under what conditions may a line be said to be in a surface?

35.2. (*a*) Graphically describe an assumed right hexagonal prism in the third angle.

(*b*) Establish and describe a line in one face of the prism and parallel to the altitude of the prism.

(*c*) Establish and describe a point on the above mentioned line.

(*d*) Is this point in the surface of the prism? Explain.

35.3. Repeat Exercise 35.2 but establish the line non-parallel to the altitude of the prism.

35.4. (*a*) Graphically describe an assumed right circular cylinder.

(*b*) Establish and describe an element of this cylinder.

(*c*) Establish and describe a point on this element.

(*d*) Is this point in the surface of the cylinder? Explain.

35.5. (*a*) Graphically describe an assumed right circular cone in first angle.

(*b*) Establish and describe an element of this cone.

(*c*) Establish and describe a point on this element.

(*d*) Is this point in the surface of the cone? Explain.

35.6. (*a*) Graphically describe the following assumed solids:

(1) An oblique pentagonal prism in the third angle.
(2) An oblique cone with circular base in the first angle.
(3) An oblique pyramid with hexagonal base in the third angle.

(*b*) Establish and describe a line on each of the above solids.

(*c*) Establish and describe a point on each of the lines.

(*d*) Which method of establishing a line (the two-point or the point-and-direction) was employed in each of the above cases? Could the other method have been applied with equal ease in each case?

36.1. (*a*) Explain how knowledge of the fact that *a line parallel to a plane is parallel to some line in the plane* assists with the task of establishing and describing a line required to be parallel to a given plane.

(*b*) Does the above fact also assist with the problem of establishing and describing a plane required to be parallel to a given line? Explain.

36.2. (*a*) Make a drawing describing an assumed oblique plane R in the first angle and also a point A in the first angle.

(*b*) State the analysis of the problem of establishing and describing a line through A and parallel to R.

(*c*) Add the description of this line to the drawing.

36.3. (*a*) Make separate drawings describing the following planes: Q 1, R 60°, L 30°; R 3, L 30°, L 60°; S 1, R 30°, 90°; T 3, 90°, L 30°.

(*b*) Add to each of the above drawings a line parallel to the plane described.

36.4. (*a*) State the analysis of the problem of establishing and describing a plane which is parallel to a given line.

(*b*) Make separate drawings describing the following lines and in each case add the description of a plane parallel to the line.

$AB(3 + 2 + 4)\ (6 + 4 + 2)$; $CD(3 + 2 + 4)\ (6 + 2 + 2)$;
$EF(3 + 2 + 2)\ (6 + 4 + 2)$; $GH(3 + 4 + 2)\ (3 + 2 + 2)$;
$AB(3 + 2 + 4)\ (3 + 2 + 2)$; $CD(3 + 2 + 2)\ (6 + 2 + 2)$.

37.1. (*a*) Explain and illustrate the conditions under which perpendicular representations are obtained when describing lines which are perpendicular.

(*b*) State the analysis of the problem of establishing and describing a line required to be perpendicular to a given line.

37.2. (*a*) Make separate drawings describing the following lines:

$AB(3 + 2 + 4)\ (6 + 4 + 2)$; $CD(3 + 2 + 4)\ (6 + 2 + 2)$;
$EF(3 + 2 + 2)\ (6 + 4 + 2)$; $GH(3 + 4 + 2)\ (3 + 2 + 2)$;
$AB(3 + 2 + 4)\ (3 + 2 + 2)$; $CD(3 + 2 + 2)\ (6 + 2 + 2)$.

(*b*) Add to each of the above drawings the description of a line which is perpendicular in direction to the one described.

(*c*) Is the line added in each of the above cases the only line in space which would meet the requirements of the problem?

(*d*) Why were the particular lines shown selected for the solutions?

(*e*) Do the lines described by the above drawings intersect? Explain.

38.1. (*a*) If a line is perpendicular to a plane in space, what is its direction relative to the traces of the plane?

(*b*) Explain why the H-representation of such a line must be perpendicular to the H-trace of the plane.

(*c*) Does this relation hold true for all corresponding representations of the line and traces of the plane?

38.2. (*a*) State the analysis of the problem of establishing and describing a line required to be perpendicular to a given plane.

(*b*) Make separate drawings describing the following planes: Q 1, R 60°, L 30°; R 3, L 30°, L 60°; S 1, R 30°, 90°; T 3, 90°, L 30°.

(*c*) Add to each of the above drawings the description of a line perpendicular to the plane.

38.3. (*a*) State the analysis of the problem of establishing and describing a plane required to be perpendicular to a given line.

(*b*) Make separate drawings of the following lines and in each case add the description of a plane perpendicular to the line.

$AB(3 + 2 + 4)\ (6 + 4 + 2)$; $CD(3 + 2 + 4)\ (6 + 2 + 2)$;
$EF(3 + 2 + 2)\ (6 + 4 + 2)$; $GH(3 + 4 + 2)\ (3 + 2 + 2)$;
$AB(3 + 2 + 4)\ (3 + 2 + 2)$; $CD(3 + 2 + 2)\ (6 + 2 + 2)$.

38.4. (*a*) State the analysis of the problem of establishing and describing a plane which is required to be perpendicular to a given line and to contain a given point.

(*b*) Make separate drawings describing the lines given in Exercise 38.3 and add the description of an assumed point (not on the line) to each drawing.

(*c*) Add to each of the above drawings the description of a plane containing the point and perpendicular to the line.

38.5. Repeat Exercise 38.3 but take the given lines in third angle (change + signs to −).

38.6. Repeat Exercise 38.4 but take the given lines and points in third angle (change + signs to −).

39.1. (*a*) Make a drawing describing an oblique plane R in the first angle.

(*b*) Add to this drawing the description of a plane S which is parallel to R.

(*c*) State and illustrate the geometric truth which is the basis of the scheme employed in establishing the plane S.

39.2. Make drawings describing the following pairs of parallel oblique planes.

(*a*) R in first angle, S in second angle.
(*b*) T in third angle, U in first angle.
(*c*) R in fourth angle, S in second angle.
(*d*) T in second angle, U in third angle.

39.3. (*a*) State the analysis of the problem of establishing and describing a plane which is required to be parallel to a given plane and to contain a given point.

(*b*) Make a drawing describing an assumed oblique plane in first angle and also an assumed point in first angle.

(*c*) Add to this drawing the description of a plane which is parallel to the given plane and contains the given point.

39.4. (*a*) Make separate drawings describing the following planes:

(1) Q 1, 90°, R 30°; (2) R 3, L 30°, 90°; (3) S 3, 90°, 90°; (4) T 1, L 45°, 90°; (5) + 3, R 30°, L 35°; (6) V 2, L 30°, R 45°.

(*b*) Add to each of the above drawings the description of an assumed point.

(*c*) Add to each of the above drawings the description of a plane parallel to the given plane and containing the given point.

39.5. Repeat Exercise 39.4 but assume the points as follows: (1) third angle; (2) first angle; (3) second angle; (4) second angle; (5) fourth angle; (6) fourth angle.

40.1. (*a*) State the analysis of the problem of establishing and describing a plane which is required to be perpendicular to a given plane.

(*b*) Make a drawing describing an oblique plane R in the first angle.

(*c*) Add to this drawing the description of a plane S perpendicular to H and to the plane R.

40.2. (*a*) Make a drawing describing an oblique plane R in the third angle and a point A in the first angle.

(*b*) Add to this drawing the description of a plane S which contains the point A and is perpendicular to H and also to the plane R.

(*c*) How does the drawing show that the plane S contains the point A?

40.3. (*a*) Make a drawing describing the points A and B in the first angle.

(*b*) Add the description of an oblique plane R containing the point A but not the point B.

(*c*) Add the description of a line BC which passes through B and is perpendicular to the plane R.

(*d*) Add the description of a plane S which contains the line BC and is perpendicular to H. What relation has the plane S to the plane R?

(*e*) Add the description of a plane T which contains the line BC and is perpendicular to V. What relation has the plane T to the plane R?

(*f*) Add the description of an oblique plane U which contains the line BC. What relation has U to R?

40.4. (*a*) Make a drawing describing two non-parallel lines, AB in the first angle and CD in the third.

(*b*) Add the description of a plane R containing AB and parallel to CD.

(*c*) Add the description of a line EF in the third angle and perpendicular to the plane R. What relative direction has the line EF to the lines AB and CD?

(*d*) Add the description of an oblique plane S which contains the line EF and which is perpendicular to the line AB. What relation has this new plane to the plane R?

(*e*) Assume a new reference plane parallel to the line CD and forming a new pair of reference planes with V as its mate. Show the representations of the lines CD and EF on this plane. Why are these representations perpendicular to each other?

QUESTIONS ON CHAPTER VII

41.1. (*a*) What position relative to a reference plane must a line occupy in order that its true length may be represented on that plane?

(*b*) What positions relative to a reference plane must intersecting lines occupy in order that the true angle between the lines may be represented on that plane? Give an exception to this rule.

(*c*) If the positions of lines which are employed to define required dimensions, are not primarily established in such positions as to enable their true lengths and angles to be represented, what two methods are available for determining the true values of the dimensions?

42.1. Name the five features involved in the rotation of a point and with the aid of a sketch explain their relations to each other.

42.2. Explain with the aid of a sketch how the five features of point rotation are represented when the axis of revolution is perpendicular to the reference plane.

42.3. Explain with the aid of a sketch how the five features of point rotation are represented when the axis of rotation is parallel to a reference plane.

42.4. Explain with the aid of a sketch how the five features of point rotation are represented when the axis of rotation is oblique to a reference plane.

43.1. (*a*) Explain and illustrate with rough sketches showing primary representations, the three characteristic arrangements of the axis of rotation relative to the primary reference planes.

(*b*) Which of these arrangements is known as the *ideal arrangement?* Why?

43.2. (*a*) Graphically describe a complete revolution of a point assuming the axis to be ideally arranged with respect to the primary reference planes.

(*b*) Name the five features of this rotation in terms of the letters assigned them on the above drawing.

(*c*) What would have been the procedure of making the above drawing had the axis been parallel to one primary reference plane and oblique to the other?

43.3. (*a*) Graphically describe a complete revolution of a point assuming the axis of rotation to be parallel to one primary reference plane and oblique to the other.

(*b*) What would have been the procedure of making the above drawing had the axis been oblique to both primary reference planes?

43.4. Graphically describe a complete revolution of a point assuming the axis of rotation to be oblique to both primary reference planes.

45.1. If a point is to be rotated 180° about an axis which is parallel to one primary reference plane and oblique to the other, is it necessary to establish a new pair of reference planes in accordance with the ideal arrangement before description of the rotation is possible? Explain.

45.2. (*a*) Graphically describe by the full method, the rotation of a point through 180° about an axis parallel to H and oblique to V.

(*b*) Graphically describe the above rotation by an abbreviated method and explain this method fully.

(*c*) Could the shorter method be applied if the angle through which the point was rotated was other than 180°? Explain.

45.3. (*a*) Graphically describe by the full method, the rotation of a point through 180° about an axis which is oblique to both V and H.

(*b*) Graphically describe the above rotation by an abbreviated method.

(*c*) Could the shorter method be used if the angle through which the point was rotated was other than 180°? Explain.

45.4. Repeat Exercise 45.2 but assume the rotational data in the third angle.

45.5. Repeat Exercise 45.3 but assume the rotational data in the third angle.

46.1. (*a*) Graphically describe an assumed axis AB in the first angle parallel to H and oblique to V also an assumed point in the first angle.

(*b*) Add to the above drawing the description of the point after it has been rotated about the axis AB until its radius of rotation is parallel to H. Use the full method of description.

(*c*) Describe the above rotation by a shorter method. Indicate clearly how all necessary dimensions are obtained.

46.2. (*a*) State concisely a rule for determining the true length of the radius of rotation of a point when the axis is parallel on one primary plane and oblique to the other.

(*b*) Demonstrate the truth of this rule using a drawing in which an additional reference plane is employed for establishing an ideal arrangement.

(*c*) Does this rule apply if the axis is oblique to both V and H?

46.3. Repeat Exercise 46.1 but assume all data in the third angle.

46.4. (*a*) Graphically describe an assumed axis AB in the first angle parallel to H and oblique to V, also a point in the first angle.

(*b*) Add to the above drawing the description of the point after it has been rotated until its radius of rotation is parallel to V. Use the full method.

(*c*) Graphically describe the above rotation by a shorter method.

46.5. Repeat Exercise 46.4 but assume the data in third angle.

46.6. (*a*) Graphically describe an assumed axis AB in the first angle oblique to both V and H, also a point in the first angle.

(*b*) Add to the above drawing the description of the point after it has been rotated until its radius of rotation is parallel to H. Use the full method.

(*c*) Graphically describe the above rotation by a shorter method.

46.7. Repeat Exercise 46.6 but assume all data in the third angle.

46.8. (*a*) Graphically describe an axis AB in the first angle oblique to V and H, also a point in the first angle.

(*b*) Add to the above drawing the description of the point after it has been rotated until it is as near as possible to H.

46.9. Repeat Exercise 46.8 but rotate the point as near V as possible.

46.10. Repeat Exercise 46.8 but assume all data in the third angle.

47.1. (*a*) Why is it sufficient to consider only the new positions of the end points when rotating a straight line?

(*b*) How would an irregularly curved line be rotated?

(*c*) When one of two intersecting lines is revolved about the other as an axis, how does the rotation of the point at which the lines intersect differ from the rotation of all other points on the revolved line? Consider each of the five features of point rotation in this explanation.

47.2. (*a*) Graphically describe an assumed line, AB, in the first angle oblique to H and V.

(*b*) How many characteristic arrangements of an axis of rotation about which to revolve the above line are possible? Explain.

(*c*) Make three drawings describing the above line after it has been rotated approximately 60° about an assumed axis. Assume the axis in a different characteristic position with respect to the primary reference planes for each of these drawings.

47.3. Repeat Exercise 47.2 but assume all data in the third angle.

48.1. (*a*) Explain and illustrate the application of the cone method of finding the true length of a line.

(*b*) Show that this method agrees with the *triangle* method of determining the true length of a radius of rotation.

48.2. (*a*) Graphically describe an assumed line AB in the first angle, oblique to H and V.

(*b*) Use the cone method to describe the true length of this line on the V-plane. Show the representations of the axis and of all necessary centers of rotation.

(*c*) On a separate drawing and again using the cone method show the true length of this line on H. Show the representations of the axis of rotation and of all necessary centers of rotation.

(*d*) It is customary to represent axes and centers as on the above drawings? Why?

48.3. May the cone method of finding the true length of a line be applied in connection with a new pair of reference planes? Illustrate.

49.1. How are the rotated positions of planes determined? Explain fully.

50.1. (*a*) Why is the problem of rotating a plane in space into a reference plane frequently encountered?

(*b*) Explain briefly how this is accomplished.

50.2. (*a*) Graphically describe an assumed oblique plane R in the first angle with traces toward the left.

(*b*) Add to this drawing using the full method, the description of this plane after it has been rotated into V about its V-trace.

(*c*) On a separate drawing and again using the full method show the description of the plane R after it has been rotated into H about its H-trace.

(*d*) Repeat each of the above exercises but use the shorter method.

50.3. Repeat Exercise 50.2 but assume all data in the third angle.

51.1. (*a*) Explain the general scheme of describing plane figures and solids in rotated positions.

(*b*) What two facts enable this full procedure to be considerably shortened?

51.2. (*a*) Graphically describe an assumed right rectangular prism in the first angle with its bases parallel to H and one face parallel to V.

(*b*) To the right of the above drawing make a second drawing describing this solid after it has been rotated approximately 45° clockwise about an axis perpendicular to V. Show the representations of the assumed axis.

(*c*) To the right of the second drawing add a third describing the prism after it has been rotated from its second position 30° counter-clockwise about an axis perpendicular to H.

(*d*) Could new pairs of reference planes be established which would bear the same relation to the prism in its original position as the primary planes bear to the prism in each of its rotated positions? Illustrate.

51.3. Repeat Exercise 51.2 but assume all data in the third angle.

51.4. (*a*) Graphically describe an assumed right square pyramid in the first angle with base parallel to H and one edge parallel to V.

(*b*) Describe the pyramid after it has been rotated 30° about an axis through its apex and oblique to V and H.

QUESTIONS ON CHAPTER VIII

52.1. Define the following in terms of the descriptive unit:

(*a*) The intersection of two lines.

(*b*) The intersection of a line with a surface.

(*c*) The intersection of two surfaces.

52.2. Why is the basic method of determining the description of an intersection founded upon a method of establishing a pair of intersecting lines?

52.3. (*a*) How many pairs of intersecting lines must be established when determining the description of the intersection of a line with a surface?

(*b*) What requirement must be satisfied in order that two lines may intersect?

(*c*) What requirement must be satisfied in order that a point, created by the intersection of two lines, may be common to a line and a surface?

52.4. (*a*) How many pairs of intersecting lines must be employed when determining the description of the intersection of two surfaces? Explain for intersecting planes and for intersecting curved surfaces.

(*b*) What requirement must be satisfied in order that a point created by the intersection of two lines may be common to two surfaces?

(*c*) What requirement must be satisfied in order that two lines may intersect?

52.5. (*a*) Explain how to establish a pair of intersecting lines so that their point of intersection may be the point at which a given line intersects a given plane.

(*b*) Explain how to establish a pair of intersecting lines so that their point of intersection may be one point of the line along which two given surfaces intersect.

52.6. When establishing a pair of lines for the purpose of determining the description of one point on the intersection of two surfaces, why should each of the surfaces contain one of the two lines? Why should these lines lie in the same plane or other surface?

52.7. (*a*) State the general scheme employed for determining a point on the intersection of two planes.

(*b*) Since it is necessary to determine the intersection of a secondary pair of surfaces during the procedure of applying this scheme, the scheme appears impractical. Explain briefly why this scheme may be successfully employed.

53.1. (*a*) Why is the intersection of two planes the easiest of all intersections to describe?

(*b*) What data must be determined in order to describe the intersection of two planes?

53.2. Explain with illustrations how the reference planes assist with the task of establishing pairs of intersecting lines which create points on the intersection of planes in space.

54.1. (*a*) What data must be determined in order to fix the intersection of two planes in space?

(*b*) How is the direction of a line usually established?

54.2. (*a*) State the analysis of the problem of determining the description of a line along which two planes intersect when the corresponding traces of the planes meet within the limits of the drawing.

(*b*) Illustrate the application of this analysis, assuming the data for the problem.

54.3. (*a*) State the analysis of the problem of determining the description of a line along which two planes intersect when one pair of corresponding traces are parallel and the other pair intersect within the limits of the drawing.

(*b*) Illustrate the application of this analysis, assuming the data for the problem.

54.4. (*a*) State the analysis of the problem of determining the description of a line along which two planes intersect when one plane is oblique to V and H and the other is parallel to either V or H.

(*b*) Illustrate the application of this analysis, assuming the data for the problem.

54.5. (*a*) Describe the following two oblique planes: *R* 1, 6, 45° *R*, 30° *R*; *S* 1, 8, 30° *L*, 60° *L*.

(*b*) Add to the above drawing, the description of the line along which the above planes intersect.

54.6. (*a*) Describe the following two oblique planes: *R* 1, 6, 30° *R*, 30° *R*; *S* 1, 8, 60° *R*, 45° *R*.

(*b*) Add to the above drawing the description of the line along which the two planes intersect.

54.7. (*a*) Describe the following two planes: *R* 1, 6, 45° *R*, 90°; *S* 1, 8, 90°, 45° *L*.

(*b*) Add to the above drawing the description of the line along which the two planes intersect.

54.8. (*a*) Describe the following two planes: *R* 1, 6, 60° *L*, 45° *R*; *S* 1, parallel to ground line.

(*b*) Add to the above drawing the description of the line along which the two planes intersect.

54.9. (*a*) Describe the following two planes: *R* 1, 6, 45° *L*, 30° *L*; *S* 3, 8, 45° *R*, 60° *R*.

(*b*) Add to the above drawing the description of the line along which the planes intersect.

54.10. (*a*) Describe the following two planes: *R* 3, 6, 45° *L*, 30° *L*; *S* 1, 8, 45° *L*, 60° *L*.

(*b*) Add to the above drawing the description of the line along which the planes intersect.

54.11. (*a*) Describe the following two planes: *R* 1, 6, 60° *L*, 45° *R*; *S* 1, 8, 60° *L*, 45° *L*.

(*b*) Add to the above drawing the description of the line along which the two planes intersect.

54.12. (*a*) Describe the following two planes: *R* 1, 6, 45° *R*, 60° *L*; *S* 1, 8, 45° *L*, 60° *L*.

(*b*) Add to the above drawing the description of the line along which the planes intersect.

54.13. (*a*) Describe the following two planes: *R* 1, 6, 90°, 45° *R*; *S* 1, 8, 90°, 45° *L*.

(*b*) Add to the above drawing the line along which the planes intersect.

54.14. (*a*) Describe the following two planes: *R* 1, 6, 45° *R*, 90°; *S* 1, 8, 45° *L*, 90°.

(*b*) Add to the above drawing the line along which the planes intersect.

54.15. (*a*) Describe the following two planes: *R* 4, 6, 60° *L*, 45° *L*; *S* 2, 8, 60° *R*, 45° *L*.

(*b*) Add to the above drawing the description of the line along which the planes intersect.

54.16. (*a*) Describe the following two planes: *R* 1, 6, 45° *R*, 45° *R*; *S* 1 parallel to and 2 inches from H.

(*b*) Add to the above drawing the description of the line along which the two planes intersect.

54.17. (*a*) Describe the following two planes: *R* 1, 6, 30° *L*, 45° *R*; *S* 1 parallel to and 2 inches from H.

(*b*) Add to the above drawing the description of the line along which the two planes intersect.

54.18. (*a*) Describe the following two planes: *R* 1, 6, 60° *R*, 30° *L*; *S* 1, parallel to and 2 inches from V.

(*b*) Add to the above drawing the description of the line along which the two planes intersect.

54.19. (*a*) Describe the following two planes: *R* 4, 6, 30° *R*, 60° *R*; *S* 4, parallel to and 2 inches from V.

(*b*) Add to the above drawing the description of the line along which the two planes intersect.

55.1. (*a*) What two methods are available for determining a straight line?

(*b*) Explain how these methods are applied to the solution of the three fundamental problems in plane intersection.

55.2. (*a*) Explain, giving illustrations, why plane intersection problems other than the three fundamental types, are in existence.

(*b*) Why must the draftsman choose between the *two-point* and the *point-and-direction* methods of determining a straight line when planning the solutions of these problems?

55.3. (*a*) Explain how the general run of plane intersection problems may differ from the three fundamental types.

(*b*) What factors are effective in causing this difference? Illustrate.

55.4. (*a*) Why is the first step of determining the description of a plane intersection concerned with establishing one point on the line of intersection?

(*b*) After establishing this point what decision must the draftsman make in respect to further procedure?

55.5. (*a*) How may a point on the intersection of two planes be determined if neither pair of traces intersect within the limits of the drawing?

(*b*) Illustrate part (*a*) assuming the necessary data.

55.6. (*a*) Describe an assumed pair of intersecting planes such that their traces, although not parallel, do not intersect within the limits of the drawing.

(*b*) Demonstrate how a point on the intersection of these planes may be determined by using an auxiliary plane parallel to H.

(*c*) Using a second copy of the original data, demonstrate how a point on the line of intersection may be determined by using an oblique auxiliary plane.

(*d*) Using a third copy of the original data, demonstrate how a point on the line of intersection may be determined by using a new reference plane.

55.7 (*a*) What requirements must be met by an auxiliary plane which is used for the purpose of determining a point on the intersection of two given planes?

(*b*) Why may new reference planes be used for the above mentioned purpose?

55.8. (*a*) What type of auxiliary plane proves most convenient to use when determining a point on the line of intersection of two given planes? Explain fully.

(*b*) When determining a line of intersection by the two-point method, should the points used be close to each other or as far apart as possible? Explain.

55.9. (*a*) Assume and describe two intersecting planes such that their traces, although not parallel, do not intersect within the limits of the drawing.

(*b*) Demonstrate how the direction of the intersection of these planes may be determined.

56.1. (*a*) What requirement, in addition to the description of in-space relations of points, lines, and surfaces, must be met by students' drawings?

(*b*) Why is it necessary that the student should distinguish between given and required information when executing drawings?

(*c*) How is this distinction accomplished?

56.2. (*a*) How are extended portions of given lines and planes represented on drawings?

(*b*) How are assumed lines and planes represented?

56.3. (*a*) Illustrate the structure of a construction line.

(*b*) For what purpose are construction lines used?

56.4. Why is it desirable to assign notation to points in the sequence of determining them when executing a solution?

57.1. (*a*) Describe the following two planes: R 1, 6, 30° R, 45° R; S 1, 8, 45° L, 60° R.

(*b*) Assume that the V-traces of the above planes fail to intersect within the limits of the drawing. Add to the above drawing the description of the line along which these planes intersect using the two-point method for determining the line.

57.2. Repeat Exercise 57.1 but determine the line by the point and direction method.

57.3. (*a*) Describe the following two planes: R 1, 6, 45° R, 60° R; S 1, 8, 60° L, 45° R.

(*b*) Assume that the V-traces of the planes fail to meet within the limits of the drawing. Add to the above drawing the description of the line along which the planes intersect using the two-point method for determining the line.

57.4. Repeat Exercise 57.3 but use the point-and-direction method for determining the line.

57.5. (*a*) Describe the following two planes: R 1, 6, 60° R, 45° R; S 1, 10, 75° R, 60° R.

(*b*) Assume that neither set of corresponding traces of these planes meet within the limits of the drawing. Add to the above drawing the description of the line along which the planes intersect using the two-point method for determining the line.

57.6. Repeat Exercise 57.5 but use the point-and-direction method for determining the line.

57.7. (*a*) Describe the following two planes: R 1, 6, 75° R, 60° R; S 1, 8, 60° R, 45° R.

(*b*) Assume that neither set of corresponding traces of these planes intersect within the limits of the drawing. Add to the above drawing the description of the line along which the planes intersect, using the two-point method for determining the line.

57.8. Repeat Exercise 57.7 but use the point-and-direction method for determining the line.

57.9. (*a*) Describe the following two planes: R 1, 6, 45° R, 60° R; S 1, 6, 60° R, 45° R.

(*b*) Add to the above drawing the description of the line along which the planes intersect, using the two-point method for determining the line.

57.10. Repeat Exercise 57.9 but determine the line by the point-and-direction method.

57.11. (*a*) Describe the following two planes: R 4, 6, 45° R, 60° L; S 2, 6, 60° R, 45° L.

(*b*) Add to the above drawing the description of the line along which the two planes intersect, using the two-point method for determining the line.

57.12. Repeat Exercise 57.11 but determine the line by the point-and-direction method.

57.13. (*a*) Describe the following two planes: R 1, traces parallel to G. L., VR 2 inches and HR 4 inches from the G. L.; S 1, traces parallel to G. L., VS 4 inches and HS 2 inches from the G. L.

(*b*) Add to the above drawing the description of the line along which the two planes intersect, using the two-point method for determining the line.

(*c*) Would the point-and-direction method be better suited to the solution of this problem?

58.1. (*a*) How is the point at which a line in space intersects a plane in space created?

(*b*) What general procedure for determining such points is suggested by the answer to the above question?

58.2. (*a*) Explain the requirements of a pair of lines in order that they may intersect to create the point at which a given line intersects a given plane.

(*b*) How are such a pair of lines established?

58.3. When selecting an auxiliary plane to be passed through a given line so as to cut a second line from a given plane, what type of plane is the more desirable, an oblique plane or a plane which is perpendicular to one reference plane? Explain fully.

58.4. (*a*) Describe the following line and plane: $AB(4 + 3 + 3)$ $(6 + 2 + 2)$; R 1, 5, 45° R, 45° R.

(*b*) Add to the above drawing the description of the point at which the given line intersects the given plane.

58.5. (*a*) Describe the following line and plane: $AB(6 + 2 + 2)$ $(6 + 3 + 3)$; R 1, 5, 45° R, 45° R.

(*b*) Add to the above drawing the description of the point at which the line intersects the plane.

58.6. (*a*) Describe the following line and plane: $AB(6 + 2 + 2)$ $(8 + 2 + 2)$; R 1, 7, 30° R, 90°.

(*b*) Add to the above drawing the description of the point at which the line intersects the plane.

58.7. (*a*) Describe the following line and plane: $AB(6 + 2 + 2)$ $(6 + 3 + 3)$; R 1, traces parallel to G. L., VR 4 inches and HR 4 inches from G. L.

(*b*) Add to the above drawing the description of the point at which the line intersects the plane.

58.8. (*a*) Describe the following line and plane: $AB(6-2+3)$ $(8-1+2)$; R 1, 10, 60° R, 30° L.

(*b*) Add to the above drawing the description of the point at which the line intersects the plane.

59.1. (*a*) How are representations created?

(*b*) Make a pictorial sketch showing how a line in space is represented upon a plane in space and how this representation is in turn represented upon the reference planes.

59.2. (*a*) Describe an assumed plane and an assumed point not in the plane.

(*b*) Add to the above drawing the description of the line which passes through the point and is perpendicular to the plane.

(*c*) Add to the above drawing the description of the point at which the line intersects the plane.

(*d*) Could the point of intersection described in part (*c*) be called the representation of the assumed point upon the assumed line?

59.3. (*a*) Describe the following line and plane: $AB(6+2+2)$ $(8+3+3)$; R 1, 7, 60° R, 60° R.

(*b*) Add to the above drawing the description of the representation of the line upon the plane.

59.4. (*a*) Describe the following line and plane: $AB(6+3+3)$ $(7+2+2)$; R 1, $6\frac{1}{2}$, 45° R, 45° R.

(*b*) Add to the above drawing the description of the representation of the line upon the plane.

59.5. (*a*) Assume the description of a line and an oblique plane such that the H-representation of the line is perpendicular to the H-trace of the plane and the V-representation of the line is parallel to the G. L.

(*b*) Add to the above drawing the description of the representation of the line upon the plane. Use the shortest procedure.

60.1. (*a*) Upon what factors is the character of the line along which a plane intersects the surface of a solid dependent?

(*b*) Illustrate with sketches the general types of lines which may be cut from prisms, pyramids, cones, and cylinders by planes.

60.2. (*a*) How are points on the line of intersection of a plane with a solid created?

(*b*) Explain the general scheme of determining and describing such points.

(*c*) What conditions enable this scheme to be successfully applied?

61.1. What two factors determine the ease with which a line cut from the surface of a solid may be described? Give illustrations.

61.2. (*a*) Why are so-called *simple lines* limited to straight lines and circles?

(*b*) What position must a circle occupy relative to the reference planes in order to be classed as a simple line?

61.3. (*a*) Why is the selection of auxiliary planes which are used in determining points on lines of intersection, limited to those which cut simple lines?

(*b*) Why is it desirable that these auxiliary planes be perpendicular to a reference plane?

61.4. (*a*) What is the most desirable arrangement of a solid relative to a pair of reference planes when it is required to describe its intersection with a plane?

(*b*) If this arrangement does not exist with reference to the primary reference planes can it be established by adding new reference planes? Explain.

62.1. (*a*) Copy Fig. 62.1 and add the description of a plane which would cut straight lines from the surface of the prism parallel to its lateral edges.

(*b*) Add the description of the lines cut by the plane. Use the full point-and-direction method for determining the lines.

62.2. (*a*) Copy Fig. 62.2 and add the description of a plane which would cut straight lines from the surface of the prism parallel to its lateral edges.

(*b*) Add the description of the lines cut by the plane. Use the full point-and-direction method for determining these lines.

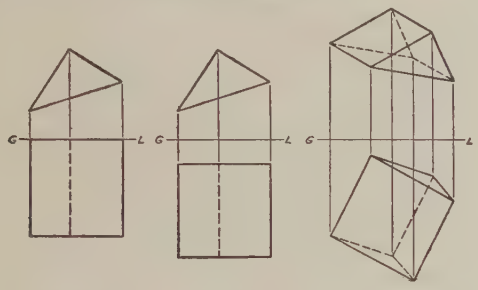


Fig. 62.1. Fig. 62.2. Fig. 62.3.

62.3. (*a*) Copy Fig. 62.3 and add the description of a plane which would cut straight lines from the surface of the prism parallel to its lateral edges.

(*b*) Add the description of the lines cut by the plane. Use the full point-and-direction method for determining the line.

62.4. (*a*) Copy Fig. 62.3 and establish a new pair of reference planes which are orthometrically positioned relative to the prism.

(*b*) Using only the new pair of reference planes add the description of a plane which would cut straight lines from the surface of the prism parallel to its lateral edges. Determine the descriptions of the lines cut by this plane.

(*c*) Complete the drawing by adding the primary descriptions of the plane and lines.

63.1. (*a*) Copy Fig. 62.2 and add the description of a plane which would cut straight lines from the surface of the prism parallel to its lateral edges.

(*b*) Add the descriptions of the lines cut by the plane. Use the shorter method.

(*c*) Explain the departures from the full procedure practiced in the above solution and justify these departures.

63.2. (*a*) Copy Fig. 62.3 and add the description of a plane which would cut straight lines from the surface of the prism parallel to its lateral edges.

(*b*) Add the descriptions of the lines cut by the plane. Use the shorter method.

(*c*) Explain the departures from the full procedure practiced in the above solution and justify these departures.

63.3. (*a*) Copy Fig. 62.3 and add a new pair of reference planes which are orthometrically arranged relative to the prisms.

(*b*) Using only the new pair of reference planes add the description of a plane which would cut straight lines from the surface of the prism parallel to its lateral edges. Determine the descriptions of the lines cut by this plane.

(*c*) Complete the drawing by adding the primary descriptions of the plane and the line cut.

63.4. (*a*) Copy Fig. 63.4 and add to each drawing the description of a plane which would cut a simple line from each.

(*b*) Could planes other than those used in the above illustrations be employed? Illustrate.

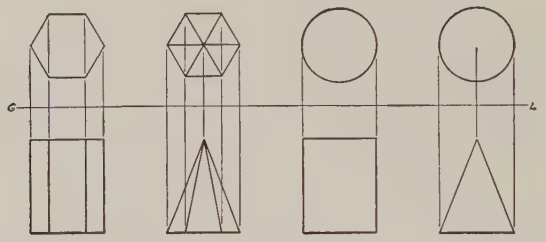


FIG. 63.4.

63.5. Copy Fig. 63.5 and add the description of the intersection of the pyramid with the plane. Use the shorter method for determining the line of intersection.

63.6. Copy Fig. 63.6 and add the description of the intersection of the cone with the plane. Use the shorter method for determining the line of intersection. (NOTE: A cone may be considered to be a pyramid having an infinite number of lateral faces.)

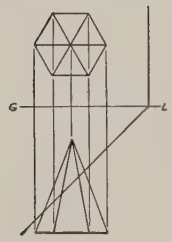


FIG. 63.5.

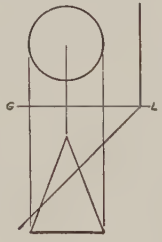


FIG. 63.6.

64.1. (*a*) Explain the procedure of planning the solution of the general type of plane-solid intersection problem.

(*b*) Is the procedure of executing a solution the same in all cases as that followed when planning it? Explain.

64.2. (*a*) What is meant by *preferred simple lines?*

(*b*) How does the task of selecting preferred simple lines differ in problems involving prisms and pyramids from that in problems involving cylinders and cones?

64.3. Copy Fig. 64.3 and add the description of the intersection of the plane and cylinder. Follow the outline given below:

(1) Test the given data to determine if the bases of the cylinder intersect the plane.

(2) Select the direction of the auxiliary planes so they will cut simple lines from the cylinder and desirable lines from the plane.

(3) Select the number and position of the preferred simple lines to be cut by the auxiliary planes.

(4) Execute the drawing which describes the intersection.

64.4. Copy Fig. 64.4 and add the description of the intersection of the plane and the prism. Follow the outline given below:

(1) Test the given data to determine if the plane intersects the bases of the prism.

(2) Select the direction of the auxiliary planes so they will cut simple lines from the prism and desirable lines from the plane.

(3) Select the number and the position of the preferred simple lines to be cut by the auxiliary planes.

(4) Execute the drawing which describes the intersection.

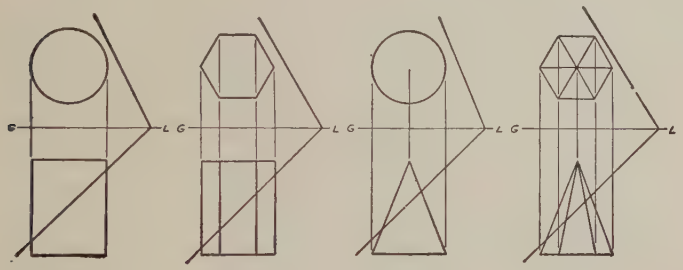


Fig. 64.3. Fig. 64.4. Fig. 64.5. Fig. 64.6.

64.5. Copy Fig. 64.5 and add the description of the intersection of the plane and cone. Follow the outline given below:

(1) Test the given data to determine if the plane intersects the base of the cone.

(2) Select the direction of the auxiliary planes so they will cut simple lines from the cone and desirable lines from the plane.

(3) Select the number and positions of the preferred simple lines to be cut from the cone by the auxiliary planes.

(4) Execute the drawing which describes the intersection. (Note: If auxiliary planes perpendicular to the axis of the cone are employed, determine the high and low points on the intersection before selecting the preferred simple lines.

64.6. Copy Fig. 64.6 and add the description of the line of intersection. Follow the outline given below:

(1) Test the given data to determine if the plane intersects the base of the pyramid.

(2) Select the direction of passing the auxiliary planes so that they will cut simple lines from the pyramid and desirable lines from the plane.

(3) Select the number and the positions of the preferred simple lines to be cut by the auxiliary planes.

(4) Execute the drawing which describes the intersection.

64.7. Copy Fig. 64.7 and add the description of the line of intersection. Follow the outline given below:

(1) Determine the high and low points on the line of intersection.

(2) Select the direction of the auxiliary planes so that they will cut simple lines from the sphere and desirable lines from the plane.

(3) Select the number and positions of the preferred simple lines to be cut by auxiliary planes.

(4) Execute the drawing describing the intersection.

64.8. Copy Fig. 64.8 and add the description of two points on the line of intersection. Follow the outline given below:

(1) Select the direction of passing auxiliary planes so that they will cut simple lines from the torus and desirable lines from the plane.

(2) Demonstrate how points on the line of intersection are described by establishing one such auxiliary plane and describing the lines it cuts from the torus and the plane.

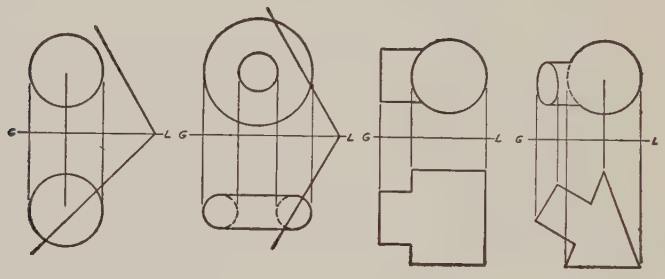


Fig. 64.7. Fig. 64.8. Fig. 65.1. Fig. 66.3.

65.1. (*a*) How are the points which make up the line along which two solids intersect, determined?

(*b*) Copy Fig. 65.1 and demonstrate the steps in the procedure of determining the description of one point on the intersection of the solids.

66.1. (*a*) What is the first requirement which must be met when devising a scheme for determining the description of a solid intersection?

(*b*) How does the above requirement limit the types of lines which may be cut from the solids by auxiliary planes?

(*c*) What relation of auxiliary plane to the reference planes is most desirable in meeting this requirement?

66.2. (*a*) Illustrate combinations of the following solids so arranged relative to each other and to the reference planes that simple lines may be cut from their surfaces by auxiliary planes perpendicular to a primary reference plane: (1) a cylinder and prism; (2) two prisms; (3) a cone and cylinder; (4) a pyramid and prism; (5) a sphere and a cone.

(*b*) Indicate for each of the above combinations of solids the type of auxiliary plane which would be used for determining points on the line of intersection.

66.3. (*a*) If due to the arrangement of a combination of solids relative to the primary reference planes, it is impossible to cut simple lines from their surfaces by auxiliary planes perpendicular to one of the reference planes, what

choice of procedures are available to the draftsman for determining the description of the line of intersection of the solids?

(*b*) Illustrate the application of each of these schemes by describing one point on the intersection of the solids shown in Fig. 66.3.

67.1. (*a*) When selecting points of a line of intersection with which to determine the description of the line, should the points be chosen for their capacity to fix the actual line in space or for the ability of their representations to fix the representations of the line?

(*b*) How many such points should be employed? Give reasons and illustrate both good and bad practice.

67.2. Copy Fig. 67.2 and add the description of the line along which the solids intersect. Follow the procedure outlined below:

1. Decide upon the direction of passing the auxiliary planes so as to cut simple lines from each of the solids.

2. Since both these solids have only plane faces the line of intersection will be formed of straight lines and it will be necessary to determine only those critical points at which these lines intersect. Examine the solids and ascertain the number of critical points, then notate in sequence the representations of these points which are already available.

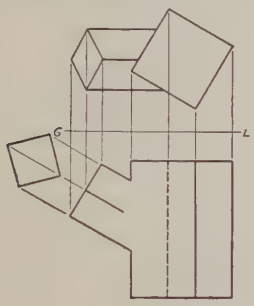


Fig. 67.2.

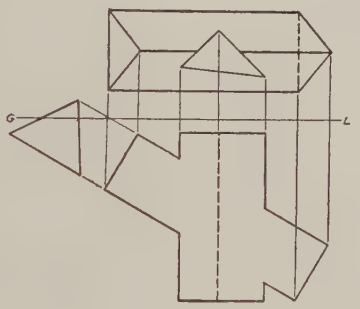


Fig. 67.3.

3. Establish, in accordance with previous plans, the auxiliary planes which cut the simple lines that intersect to create the critical points.

4. Complete the description of the line of intersection by completing the description of the critical points and by adding the representations of the straight lines which form the intersection.

67.3. Repeat Exercise 67.2 but use the combination of solids shown in Fig. 67.3 for the given data.

67.4. Copy Fig. 65.1 and add the description of the line of intersection. Follow the procedure outlined below:

1. Decide upon the direction of passing the auxiliary planes so as to cut simple lines from each of the solids.

2. Estimate the general characteristics of the representations of the line of intersection and decide upon a suitable number of preferred points to be used in accurately describing the line.

3. Establish a set of simple lines on one of the solids such that the lines may be cut by auxiliary planes as planned and also so they will pass through the preferred points on the line of intersection.

4. Establish the auxiliary planes so as to cut the already established set of simple lines, and then describe the corresponding lines cut from the second solid. Notate the representations of the intersections of corresponding simple lines and complete the description of the line of intersection by adding its representations to the drawing.

67.5. Repeat Exercise 67.4 but use the solids shown in Fig. 67.5 as the given data.

67.6. Repeat Exercise 67.4 but use the solids shown in Fig. 67.6 as the given data.

67.7. Repeat Exercise 67.4 but use the solids shown in Fig. 67.7 as the given data.

67.8. Repeat Exercise 67.4 but use the solids shown in Fig. 67.8 as the given data.

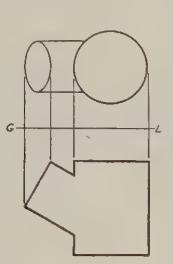


Fig. 67.5.

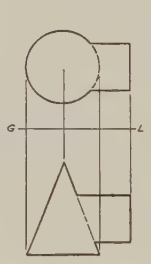


Fig. 67.6.

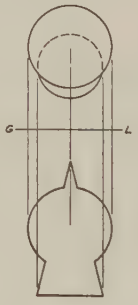


Fig. 67.7.

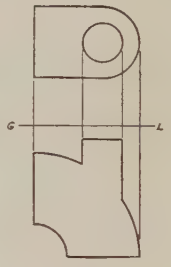


Fig. 67.8.

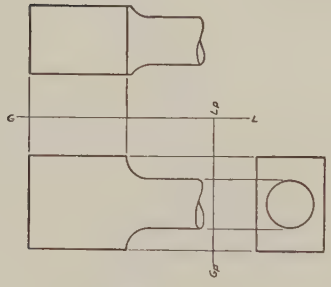


Fig. 67.9.

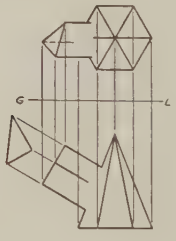


Fig. 67.10.

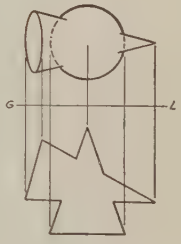


Fig. 67.14.

67.9. Repeat Exercise 67.4 but use the solids shown in Fig. 67.9 as the given data.

67.10. Copy Fig. 67.10 and demonstrate how one point on the intersection of the solids may be described by establishing a new pair of reference planes which enable the use of auxiliary planes taken perpendicular to a reference plane.

67.11. Copy Fig. 67.10 and describe one point on the intersection of the solids without using a new pair of reference planes.

67.12. Repeat Exercise 67.10 but use the solids shown in Fig. 66.3 as given data.

67.13. Copy Fig. 66.3 and describe one point on the intersection of the solids without using a new pair of reference planes.

67.14. Copy Fig. 67.14 and describe one point on the intersection of the solids without using a new pair of reference planes.

67.15. Copy Fig. 67.15 and describe one point on the line of intersection of the solids without using a new pair of reference planes. Why is the procedure of describing this point simpler than that of describing the point in Exercise 67.14?

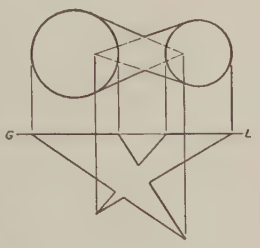


Fig. 67.15.

68.1. (*a*) Under what conditions may spherical surfaces be used as auxiliary surfaces for cutting pairs of simple lines from the surfaces of solids?

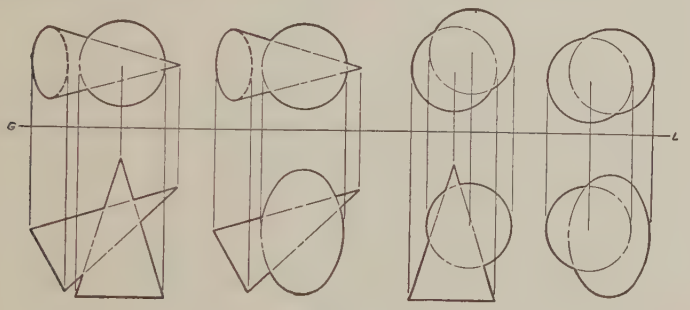


Fig. 68.1.

(*b*) Demonstrate how one point may be determined on the intersection of each of the combinations of solids shown in Fig. 68.1 by using a sphere as the auxiliary surface.

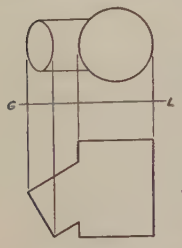


Fig. 69.1.

69.1. (*a*) Demonstrate the short practical method of establishing the description of equally spaced points and lines on the surface of the horizontal cylinder shown in Fig. 65.1.

(*b*) Repeat for the oblique cylinder shown in Fig. 69.1. Can this scheme be employed when determining the H-representation of this cylinder?

69.2. (*a*) Demonstrate the short practical method of establishing the description of equally spaced elements on the surface of the oblique cone shown in Fig. 67.14.

(*b*) Show how this method may be applied as a means of establishing the H-representation of the base of this cone.

QUESTIONS ON CHAPTER IX

70.1. Why does the initial step of describing tangent relations consist of gathering information which permits the conditions of tangency to be defined in terms of simple in-space relations of points, lines and planes?

71.1. (*a*) When is a straight line tangent to a plane curve?

(*b*) Must the line be in the plane of the curve?

(*c*) May the lines cross at the point of tangency?

71.2. (*a*) When is a straight line tangent to a curved surface?

(*b*) What is the *point of tangency?*

71.3. (*a*) When is a plane tangent to a singly curved surface?

(*b*) When is a plane tangent to a doubly curved surface?

71.4. (*a*) What is an *element of tangency?*

(*b*) When is a line *normal* to a curve?

(*c*) When is a line *normal* to a surface?

72.1. (*a*) Demonstrate and explain the fact that representations of tangent lines are tangent at the representation of the point of tangency.

(*b*) What data are required to fix a straight line?

(*c*) Since the representation of a straight line is also straight, does the same requirement of data apply to the fixation of representations of lines as to actual lines?

(*d*) Explain how the fact stated in part (*a*) assists in determining the representation of a straight line required to be tangent to a given plane curve.

72.2. (*a*) Make four drawings each describing a circle in one of the following positions:

(1) In the H-plane.
(2) Parallel to V.
(3) Perpendicular to H but oblique to V.
(4) Oblique to V and H.

(*b*) Add to each of the above drawings the description of a point on the circle.

(*c*) Add to each of the above drawings the description of a line through the point and tangent to the circle.

72.3. (*a*) Make four drawings each describing a circle in one of the following positions:

(1) In the V-plane.
(2) Parallel to the H-plane.
(3) Perpendicular to V but oblique to H.
(4) Oblique to V and H.

(*b*) Add to each of the above drawings the description of a point outside the circle but in its plane.

(*c*) Add to each of the above drawings the description of a line through the point and tangent to the circle.

(*d*) Is the line shown in each case the only line which would satisfy the requirements? Explain.

72.4. (*a*) Is it possible to draw a straight line tangent to a plane curve and through a point not in the plane of the curve? Explain.

(*b*) Is it possible to draw a straight line through a point on a radius of a curve and tangent to the curve? Explain.

72.5. (*a*) What positions relative to the primary reference planes are most desirable for a plane curve to occupy when it is required to describe a tangent to the curve? Explain.

(*b*) May the position of a given curve be changed so as to establish this desired relation between the curve and the primary planes? Illustrate.

(*c*) Illustrate how a desirable relation between a given curve and a new pair of planes may be established.

72.6. (*a*) Describe an assumed oblique circle and an assumed oblique plane both in the first angle.

(*b*) Select a point in the assumed plane and also in the plane of the circle.

(*c*) Add the description of a line drawn through the chosen point and tangent to the circle.

72.7. (*a*) Describe an assumed oblique circle and an assumed oblique line both in the third angle.

(*b*) Investigate the possibility of drawing a line through a point on the assumed line and tangent to the assumed circle.

(*c*) Reassume the data for the oblique line, if necessary, and add the description of such a tangent line.

73.1. (*a*) When is a straight line tangent to a curved surface?

(*b*) Explain the general nature of the method used for describing a straight line which is required to be tangent to a given solid and to pass through a given point.

73.2. (*a*) What is meant by a *simple line?*

(*b*) Why should simple lines be cut from solids in preference to others when determining the description of a straight line required to be tangent to a given solid and to pass through a given point?

(*c*) What function do these simple lines serve in the scheme of the solution?

73.3. (*a*) What position must a right circular cone occupy relative to a pair of reference planes in order that simple circles may be cut from its surface?

(*b*) In case a cone does not occupy this position, explain and illustrate two methods of establishing the desired relation.

73.4. (*a*) Describe a right circular cone in third angle with its axes perpendicular to H, also a point on the surface of the cone.

(*b*) Add to the above drawing the description of a line which passes through the point and is tangent to the cone.

73.5. (*a*) Describe a right circular cone in the first angle with its axis perpendicular to H, also a point in the first angle which is not on the surface of the cone.

(*b*) Add to the above drawing the description of a line which passes through the point and is tangent to the cone.

73.6. (*a*) Describe a sphere in the third angle, also a point on the surface of the sphere.

(*b*) Add to the above drawing the description of a line which passes through the point and is tangent to the sphere.

73.7. (*a*) Describe a sphere in the first angle, also a point in first angle which is not on the surface of the sphere.

(*b*) Add to the above drawing the description of a line which passes through the point and is tangent to the sphere.

74.1. (*a*) What methods are preferable for fixing the position of a plane in space?

(*b*) Why are these methods preferred?

74.2. (*a*) Describe an assumed pair of intersecting lines in the third angle.

(*b*) Add to the above drawing the description of the plane determined by the two lines.

74.3. (*a*) Describe an assumed pair of parallel lines in the first angle.

(*b*) Add to the above drawing the description of the plane determined by the two lines.

74.4. (*a*) Describe an assumed pair of lines in the first angle which are perpendicular to each other in direction.

(*b*) Add to the above drawing the description of a plane which contains one of the lines and is perpendicular to the other.

74.5. (*a*) Describe a pair of parallel lines in the third angle.

(*b*) Add the description of a plane which contains one of these lines and is parallel to the other.

74.6. (*a*) Describe a plane and a line in the first angle.

(*b*) Add to the above drawing the description of a second plane which contains the given line and is perpendicular to the given plane.

75.1. (*a*) What is the first step of devising a solution to the problem of describing a plane required to be tangent to a given solid?

(*b*) When is a plane tangent to a singly curved surface of revolution?

(*c*) What two lines may be employed for fixing the position of a plane which is tangent to a singly curved surface of revolution?

75.2. (*a*) Describe a right circular cone orthometrically positioned in the third angle and also a point on the surface of the cone.

(*b*) What two intersecting lines may be established which will determine a plane tangent to this cone and through the given point?

(*c*) Add to the above drawing the description of this tangent plane.

(*d*) Is this the only plane which will meet the conditions of the problem?

75.3. Repeat Exercise 75.2 but use a right circular cylinder orthometrically positioned in the first angle.

75.4. (*a*) Describe an oblique cone in the first angle with a circular base in the H-plane, also a point on the surface of the cone.

(*b*) What two intersecting lines may be established which will determine a plane tangent to the cone and containing the given point?

(*c*) May a trace of the required plane be employed as one of these lines? Explain.

(*d*) Add the description of the tangent plane to the above drawing.

75.5. Repeat Exercise 75.4 but use an oblique cylinder in the third angle with a circular base in the H-plane.

75.6. (*a*) Describe a right circular cone orthometrically positioned in the third angle, also a point on the surface of the cone.

(*b*) Add to the above drawing the description of the circle cut by the H-plane from the extended portion of the cone.

(*c*) Explain how this circle may be employed when establishing a plane tangent to the cone and containing the given point.

(*d*) Add the description of the tangent plane to the drawing.

75.7. (*a*) When a plane is required to be tangent to a given singly curved surface and to contain a given point not on this surface, is the element of tangency directly available for use as a determining line?

(*b*) Explain and demonstrate how the element of tangency may be established. As the given solid for this illustration use a right circular cone orthometrically positioned in the third angle.

(*c*) Complete the above drawing by adding the description of the tangent plane.

75.8. (*a*) Describe an oblique cone in the first angle with circular base in the H-plane, also a point in the first angle not on the surface of the cone.

(*b*) What determining data in addition to that directly available is required in order to fix the location of the H-trace of a plane which is tangent to the cone and contains the given point?

(*c*) Explain how these data may be established.

(*d*) Add to the above drawing the description of the tangent plane.

75.9. Repeat Exercise 75.8 but use an oblique cylinder in the third angle with a circular base in the H-plane, and a point also in the third angle.

75.10. (*a*) Explain why a line through a given point and the apex of a given cone must be a line of a plane which contains the given point and is tangent to the cone.

(*b*) Explain why a line through a given point and parallel to the elements of a given cylinder must be a line of a plane which contains the given point and is tangent to the cylinder.

75.11. (*a*) Describe roughly a cone in the third angle non-orthometrically positioned relative to the primary reference planes, also a point on the surface of the cone.

(*b*) Add to the above drawing the description of a plane which contains the point and is tangent to the cone.

75.12. (*a*) Describe roughly a cylinder in the first angle non-orthometrically positioned relative to the primary reference planes, also a point on the surface of the cylinder.

(*b*) Add to the above drawing the description of a plane which contains the given point and is tangent to the cylinder.

75.13. (*a*) What two methods may be used for describing a plane which is required to be tangent to a non-orthometrically positioned cone and which must contain a given point not on the surface of the cone?

(*b*) Demonstrate these two solutions assuming the necessary data.

76.1. What are the limitations in respect to the number of planes which may be established tangent to a doubly curved surface of revolution and which contain:

(1) A given point on the surface?
(2) A given point not on the surface?
(3) A given line which does not intersect the surface?

76.2. (*a*) When is a plane tangent to a doubly curved surface of revolution?

(*b*) What two forms of data are preferred for fixing the position of a plane in space?

(*c*) Would two straight lines tangent to a doubly curved surface at the same point determine a plane tangent to the surface at that point?

(*d*) Would a plane perpendicular to a radius of a doubly curved surface at a point on the surface be tangent to the surface?

76.3. (*a*) Describe a sphere in the third angle also a point on the surface of the sphere.

(*b*) What defining data may be employed for fixing the position of a plane which contains the given point and is tangent to the sphere?

(*c*) Add to the above drawing the description of this tangent plane.

76.4. (*a*) Describe an annular torus in the third angle with its axis perpendicular to H, also a point on the surface of the torus.

(*b*) What defining data may be employed for fixing the position of a plane which contains the given point and is tangent to the torus?

(*c*) Add to the above drawing the description of this tangent plane.

77.1. (*a*) Describe a sphere in the third angle also a point in the third angle which is not on the surface of the sphere.

(*b*) How many lines may be established through the point and tangent to the sphere?

(*c*) Add the description of a line through the point and tangent to the sphere.

(*d*) How many planes may be established tangent to the sphere at the point of tangency of this line?

(*e*) Would a plane tangent to the sphere at the point of tangency of the line, contain the line?

(*f*) Add to the drawing the description of a plane which contains the given point and is tangent to the sphere.

77.2. (*a*) Describe an annular torus in the third angle with its axis perpendicular to H, also a point in the third angle which is not on the surface of the torus.

(*b*) Add to the above drawing the description of a line through the given point and tangent to the torus. Is this the only line which could be established under the conditions?

(*c*) Would a plane tangent to the torus at the point of tangency of the line, contain the line?

(*d*) Add to the drawing the description of a plane which contains the given point and is tangent to the torus.

78.1. (*a*) How many planes may be passed through a given line and tangent to a given doubly curved surface of revolution?

(*b*) What data would be preferred for determining the position of such a plane? Explain.

78.2. (*a*) Describe a sphere in the third angle also a line in the third angle which does not intersect the sphere.

(*b*) Describe an auxiliary plane which passes through the center of the sphere and is perpendicular to the given line. Also describe the point (*G*) at which the given line intersects this auxiliary plane.

(*c*) Explain why a line through the point *G* and tangent to the great circle

cut from the sphere by the auxiliary plane, is a line of a plane which contains the given line and is tangent to the given sphere.

(*d*) Add the description of this tangent plane to the drawing.

78.3. (*a*) Describe an annular torus in the third angle with its axis perpendicular to H, also a line in the third angle which does not intersect the torus nor the axis of the torus.

(*b*) What data would determine a plane which contains the given line and is tangent to the torus? What part of these data is already available?

(*c*) Explain how the remaining portion of these data may be established.

(*d*) Add to the drawing the description of the tangent plane.

(*e*) Would the solution of this problem be simpler if the given line intersected the axis of the torus? Explain and demonstrate.

78.4. (*a*) Describe a sphere in the third angle, also a line in the third angle which intersects the vertical axis of the sphere but not the sphere itself.

(*b*) Using the general method, determine the description of a plane which is tangent to the sphere and which contains the given line.

(*c*) Explain how to solve a similar problem in which the given line does not intersect the vertical or horizontal axis of the sphere.

QUESTIONS ON CHAPTER X

79.1. (*a*) What characteristic other than the locations of their composite points may be employed for specifying the position of a line or a plane in space?

(*b*) Explain how the angle between a line and a plane is measured.

(*c*) Explain how the angle between two planes is measured.

79.2. (*a*) How must a line be arranged relative to a pair of reference planes in order that the true value of the angle between the line and one of the planes may be represented?

(*b*) Describe a line which is so positioned that the true value of its angle with the H-plane is represented.

(*c*) Describe a line which is so positioned that the true value of its angle with the V-plane is represented.

79.3. (*a*) How must a plane be arranged relative to a pair of reference planes in order that the true value of the angle between the plane and a reference plane may be represented?

(*b*) Describe a plane which is so positioned that the true value of its angle with the H-plane is represented.

(*c*) Describe a plane which is so positioned that its true angle with the V-plane is represented.

80.1. (*a*) Explain why and how a right circular cone is used when establishing a line to make a given angle with one of a pair of reference planes and to be oblique to the other.

(*b*) Describe a line which is parallel to V and which makes an angle of 60° with H.

(*c*) Add to the above drawing the description of a right circular cone generated by rotating the line about an axis through one end of the line and perpendicular to H.

(*d*) Do all the elements of this cone make the same angle with H?

(*e*) In what position is the generating line represented in its true length?

(*f*) Is the true angle between the line and the reference plane to which the axis is perpendicular also represented while in this position?

80.2. (*a*) Describe a right circular cone every element of which makes an angle of 45° with the V-plane.

(*b*) Describe a right circular cone every element of which makes an angle of 30° with a profile plane.

80.3. (*a*) Describe an assumed point *A* in the third angle.

(*b*) Add to the above drawing the description of a 4-inch line which passes through the point *A* and which makes an angle of 30° with H and is oblique to V.

80.4. (*a*) Describe an assumed point *A* in the first angle.

(*b*) Add to the above drawing the description of a 4-inch line which passes through the point *A* and which makes an angle of 60° with the V-plane and is oblique to H.

81.1. (*a*) Explain how and why a pair of right circular cones may be employed for establishing a line which makes a specified angle with each of a pair of reference planes.

(*b*) Must the elements of the two cones be equal in length?

(*c*) Must the cones have a common apex?

(*d*) What limitations are there to the angles a line may make simultaneously with V and H? Explain.

81.2. (*a*) Describe an assumed point *A* in the first angle.

(*b*) Add to the above drawing the description of a line 4 inches long which passes through the point *A* and which makes angles of 45° with H and 30° with V.

81.3. (*a*) Describe an assumed point *A* in the third angle.

(*b*) Add to the above drawing the description of a line 4 inches long which makes angles of 30° with H and 45° with V.

(*c*) How many positions may a line occupy and meet the requirements of making specified angles with V and H? Explain fully.

82.1. (*a*) Explain how and why a right circular cone may be employed when establishing a plane in space at a specified angle with one reference plane and oblique to the other.

(*b*) How many planes may be established to meet such a specification?

(*c*) If the plane is required to contain a given line and make a given angle with a reference plane, how many positions may it occupy?

82.2. (*a*) Describe an assumed oblique line in the third angle.

(*b*) Add to the above drawing the description of a plane which contains the given line and makes an angle of 60° with H. Use a construction cone.

82.3. (*a*) Describe an assumed oblique line in the first angle.

(*b*) Add to the above drawing the description of a plane which contains the line and makes an angle of 45° with V. Use a construction cone.

83.1. (*a*) If a plane in space is perpendicular to a line in space what relation exists between the angles made by the plane and the angles made by the line with the reference planes?

(*b*) Explain a scheme for establishing a plane at a specified angle with each of a pair of reference planes.

(*c*) How many positions may a plane occupy and satisfy the requirements of a given problem of this nature?

83.2. (*a*) Describe an assumed point in the third angle.

(*b*) Add to the above drawing the description of a plane which contains the given point and which makes angles of 60° with H and 45° with V. Establish the plane by making it perpendicular to a complementary line.

(*c*) What are the limitations to the angles that a plane may make simultaneously with a pair of reference planes?

83.3. (*a*) Describe an assumed point in first angle.

(*b*) Add to the above drawing the description of a plane which contains the given point and which makes angles of 60° with H and 45° with V. Establish the plane by making it perpendicular to a complementary line.

84.1. (*a*) Explain a general scheme by means of which a plane figure may be described when the plane of the figure is required to be perpendicular to one reference plane and oblique to the other.

(*b*) Describe an assumed point in the first angle.

(*c*) Add to the above drawing the description of a hexagon with its center at the assumed point and the plane of which makes angles of 60° with H and 45° with V. Make one edge of the hexagon at an angle of 30° with V.

84.2. (*a*) Describe an assumed point *A* in the third angle.

(*b*) Add to the above drawing the description of a circle with its center at the assumed point and the plane of which makes angles of 60° with H and 45° with V.

85.1. (*a*) Describe an assumed point *A* in the first angle.

(*b*) Add to the above drawing the description of a square with its center at the assumed point and its plane making an angle of 45° with H and any angle other than 90° with V. One side of the square is parallel to H.

85.2. (*a*) Describe an assumed point *A* in the third angle.

(*b*) Add to the above drawing the description of a hexagon which has its center at the assumed point and whose plane makes angles of 60° with H and 45° with V. One side of the hexagon makes an angle of 45° with the H-plane.

(*c*) What are the limiting angles that a side of the hexagon may make with the H-plane?

86.1. (*a*) Describe an orthometrically positioned right rectangular prism in the third angle.

(*b*) Show the successive stages in determining a description of the above prism when in such a position that its lateral edges make angles of 60° with H and 15° with V.

QUESTIONS ON CHAPTER XI

87.1. (*a*) Is it advisable to attempt to master the science of graphic description by memorizing a method of describing every possible in-space relation of points, lines, surfaces, and solids?

(*b*) What knowledge enables a draftsman to solve successfully an unfamiliar problem in graphic description?

(*c*) Explain how experience makes such solutions easier.

87.2. (*a*) Show that due to pyramiding of knowledge it is absolutely essential to be familiar with certain fundamental solutions in order to possess a working knowledge in graphic description.

(*b*) What is meant by a "draftsman's stock of solutions"?

88.1. (*a*) Is information actually created or is it simply made more available by computation?

(*b*) With what information does graphic computation deal?

(*c*) In what specific form is this information sought by graphic computation?

88.2. (*a*) What is a linear dimension?

(*b*) What is an angular dimension?

(*c*) Explain how lines may be employed to define the above dimensions.

(*d*) How must these defining lines be represented in order that the true value of the dimensions they define are also represented?

88.3. (*a*) Explain the rôle played by graphic description in graphic computation.

(*b*) How is the result of a graphic computation expressed?

88.4. Outline the general procedure of performing a graphic computation.

89.1. (*a*) Describe two assumed points in the first angle.

(*b*) Determine by a graphic computation the true distance between these points. State a formal analysis of the problem before proceeding with the solution.

89.2. (*a*) Describe two assumed points; one in the first angle and one in the third angle.

(*b*) Determine by a graphic computation the true distance between these points. State a formal analysis of the problem before proceeding with the solution.

89.3. Explain and illustrate two methods of representing the true length of a line which is oblique to the primary pair of reference planes.

90.1. (*a*) Describe an assumed line in the first angle oblique to V and H, also a point in first angle not on the line.

(*b*) Determine by a graphic computation the perpendicular distance from the point to the line. State a formal analysis of the problem before proceeding with the solution.

(*c*) Could this scheme of solution be employed for determining the distance from the point to the line measured along a line which passes through the point and makes a specified angle with the given line?

90.2. (*a*) Describe an assumed line AB in the first angle and oblique to H and V, also a point C in first angle not on the line AB.

(*b*) Add to the above drawing the description of a plane which contains the point C and is perpendicular to the line AB. Also add the description of the point F at which the assumed line AB intersects this plane.

(*c*) Would a line from the point C to the point F define the perpendicular distance from C to the line AB? Show a true length representation of this line.

90.3. (*a*) Explain two methods of establishing a line which defines the perpendicular distance from a given point to a given line.

(*b*) Could both of these methods be employed for establishing a line to define a dimension from the given point to the given line and at an angle with the given line other than 90°? Explain.

90.4. Explain and illustrate a convenient and accurate method of rotating a line and a point into a reference plane about a trace of a plane which contains them.

91.1. (*a*) Describe an oblique plane in the first angle, also a point in the first angle which is not in the plane.

(*b*) Determine by a graphic computation the perpendicular distance from the point to the plane. State a formal analysis of the problem before proceeding with the solution.

91.2. (*a*) Describe an oblique plane R in the third angle, also a point A in the third angle which is not in the oblique plane.

(*b*) Represent the plane R and the point A on a new reference plane which is taken perpendicular to R.

(*c*) Add the representation, on this new reference plane, of a line from A and perpendicular to R. Is this a true length representation of the line? Does the length of this line define the perpendicular distance from the point A to the plane R?

91.3. Explain two methods which may be employed for establishing a line which defines the perpendicular distance from a given point to a given plane.

92.1. (*a*) Describe an oblique plane in the third angle also a point in the third angle which does not lie in the plane.

(*b*) Determine by a graphic computation the vertical distance from the point to the plane. State a formal analysis of the problem before proceeding with the solution.

93.1. (*a*) Describe two non-intersecting, non-parallel lines in the third angle.

(*b*) Determine by a graphic computation the value of the shortest distance between these lines.

(*c*) Does this solution provide a description of the actual shortest line which may be established between the given lines?

93.2. (*a*) Make a rough pictorial sketch showing a plane containing one of a pair of skew lines and parallel to the other. Next, show the representation of the second skew line on this plane. Add a line perpendicular to the plane at the point of intersection of the first skew line with the representation of the other. Is this line perpendicular to both skew lines? Does this line define the shortest distance between the skew lines?

(*b*) State a formal analysis of the problem of determining the shortest distance between a pair of skew lines. Base this analysis upon the facts brought out in the above sketch.

(*c*) Describe two non-intersecting, non-parallel lines in the first angle and determine the shortest distance between these lines basing the solution upon the above analysis.

93.3. (*a*) Would the shortest line which may be drawn between a pair of skew lines be represented by its true length on a reference plane which is perpendicular to one of the skew lines?

(*b*) Describe an assumed pair of oblique skew lines in the third angle.

(*c*) Establish a new pair of reference planes such that one is perpendicular to one of the skew lines.

(*d*) Describe the skew lines on the new reference planes and add the description of their common perpendicular.

94.1. (*a*) Describe two oblique parallel planes in the third angle.

(*b*) Determine by a graphic computation the distance between these planes. State a formal analysis of the problem before proceeding with the solution.

94.2. (*a*) Describe an oblique plane in the first angle and a second plane parallel to the first in the third angle.

(*b*) Determine by a graphic computation the distance between these planes. State a formal analysis of the problem before proceeding with the solution.

95.1. (*a*) How must a pair of lines be arranged relative to a reference plane in order that the true value of the angle between the lines may be represented on that reference plane?

(*b*) Does a right angle offer an exception to the above rule? Explain.

(*c*) Briefly explain three different schemes for representing the true value of an angle when its defining lines are not favorably positioned relative to the primary reference planes.

95.2. (*a*) Describe two intersecting oblique lines in the third angle.

(*b*) Determine by a graphic computation the true value of the angle between these lines. Solve this problem by rotating the lines into the H-plane. State a formal analysis of the problem before proceeding with the solution.

95.3. (*a*) Describe two intersecting oblique lines in the third angle.

(*b*) Determine by a graphic computation the true value of the angle between these lines. Solve this problem by rotating the lines parallel to V. State a formal analysis of the problem before proceeding with the solution.

95.4. (*a*) Describe two intersecting oblique lines in the third angle.

(*b*) Determine by a graphic computation the true value of the angle between these lines. Solve this problem by representing the lines on a new reference plane which is parallel to the plane of their angle.

96.1. (*a*) Explain with the aid of a sketch how the angle between a line and a plane is measured.

(*b*) Point out the defining lines of the angle in this sketch.

(*c*) Add a line to the sketch which intersects the first line and is perpendicular to the given plane. What relation exists between the angle this new line makes with the given line and the angle the given line makes with the given plane?

96.2. (*a*) Why is it usually expedient to determine the complement of the angle between a line and a plane rather than the angle itself?

(*b*) Which of these methods is best suited for determining the angle between a line and a reference plane? Illustrate.

96.3. (*a*) Describe an oblique plane in the third angle, also an oblique line which does not lie in the plane.

(*b*) Determine by a graphic computation the angle between the line and the plane. State a formal analysis of the problem before proceeding with the solution.

97.1. (*a*) Explain with the aid of a sketch how the angle between two intersecting planes is measured.

(*b*) Point out the defining lines of the angle in this sketch.

(*c*) What relation have these defining lines to the intersection of the planes?

97.2. (*a*) State the formal analysis of the problem of determining the angle between two intersecting planes.

(*b*) Copy Fig. 97.2 and describe the true angle between these planes. Follow the full logical procedure when executing this solution.

97.3. (*a*) State the formal analysis of the problem of determining the angle between two intersecting planes.

(*b*) Make a pictorial sketch showing that a triangular plane figure is formed by the defining lines of the angle and the trace of the plane which cuts the defining lines.

(*c*) From an examination of this figure compile a number of facts which would be of value in establishing the defining lines of the angle.

(*d*) Suggest a scheme of employing these facts in determining one defining line by the two-point method.

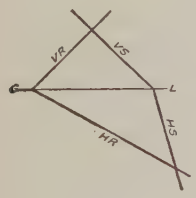


Fig. 97.2.

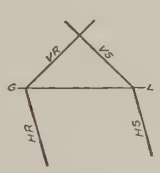


Fig. 97.7.

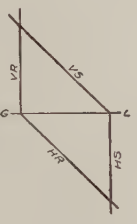


Fig. 97.8.

(*e*) Suggest a second scheme based on the point-and-direction method.

97.4. Copy Fig. 97.2 and describe the true angle between the planes. Use the point-and-direction method for establishing one defining line of the angle.

97.5. (*a*) Describe an oblique plane R in the first angle.

(*b*) State the formal analysis of the problem of determining the angle between any two planes.

(*c*) Describe the true angle between the plane R and the H-plane.

97.6. (*a*) Describe an oblique plane R in the first angle.

(*b*) State the formal analysis of the problem of determining the true angle between any two planes.

(*c*) Describe the true angle between the plane R and the V-plane.

97.7. Copy Fig. 97.7 and describe the true value of the angle between the planes.

97.8. Copy Fig. 97.8 and describe the true value of the angle between the planes.

97.9. If a new reference plane were established perpendicular to the intersection of two planes, would the angle between the traces of the planes on the reference plane be the true angle between the planes? Explain.

97.10. (*a*) Describe two intersecting planes R and S in the first angle, also a point A in the first angle.

(*b*) Add the descriptions of two lines which pass through the point A; take one line perpendicular to R and the other perpendicular to S.

(*c*) What relation exists between the angle made by the planes and the angle made by the lines?

(*d*) Describe the true value of the angle between the lines.

QUESTIONS ON CHAPTER XII

98.1. (*a*) Illustrate with a sketch the process of developing a conical surface.

(*b*) Is this process reversible?

(*c*) Is it permissible to stretch or otherwise distort a surface while developing it?

98.2. (*a*) What are the requirements of a surface in order that it may be truly developed?

(*b*) What surfaces are truly developable?

99.1. (*a*) Illustrate with a sketch the process of rolling out the development of a cylindrical surface.

(*b*) Are the elements of the cylinder parallel in their developed positions?

(*c*) Are the elements spaced at distances on the development equal to those on the original surface?

99.2. (*a*) Show by sketch how to establish a *stretch-out line* for developing a cylindrical surface.

(*b*) What is the functions of a *measuring line?* Illustrate how these lines are established.

99.3. Copy Fig. 99.3 and develop the cylindrical surface. Notate fully.

99.4. Copy Fig. 99.4 and develop the cylindrical surface. Notate fully.

99.5. Copy Fig. 99.5 and develop the cylindrical surface. Notate fully.

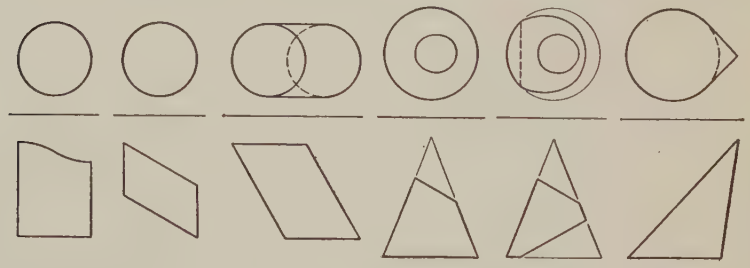


Fig. 99.3. Fig. 99.4. Fig. 99.5. Fig. 100.3. Fig. 100.4. Fig. 100.5.

99.6. (*a*) Why is it customary to cut a surface along the shortest element when drawing its development?

(*b*) Is it customary to represent the inside or the outside of a surface in its development?

100.1. (*a*) Illustrate with a sketch the process of rolling out the development of a conical surface.

(*b*) Show that all elements of the surface in their developed positions pass through the developed position of the apex.

(*c*) Assume a circumference on the conical surface and show how this circumference is rolled out on the development.

(*d*) Is the distance along this developed circumference and between a pair of developed elements the same as the distance between these elements on the original surface measured along the original circumference?

100.2. (*a*) Show by sketch how to establish a *stretch-out line* for developing a conical surface.

(*b*) What is the function of a *measuring line?* Illustrate how these lines are established.

100.3. Copy Fig. 100.3 and develop the conical surface. Notate fully.

100.4. Copy Fig. 100.4 and develop the conical surface. Notate fully.

100.5. Copy Fig. 100.5 and develop the conical surface. Notate fully.

101.1. Demonstrate the two general methods of determining the true shape of a non-orthometrically positioned plane figure?

101.2. (*a*) Copy Fig. 101.2 and determine the true shape of top face of the pyramid by representing the face on a new reference plane which is parallel to the face.

(*b*) Show how this scheme is modified to suit the needs of practical work.

101.3. (*a*) Copy Fig. 101.3 and determine the true shape of the top face of the cylinder by rotating this face into a reference plane about a trace of the plane of the face.

(*b*) Show how this scheme is modified to suit the needs of practical work.

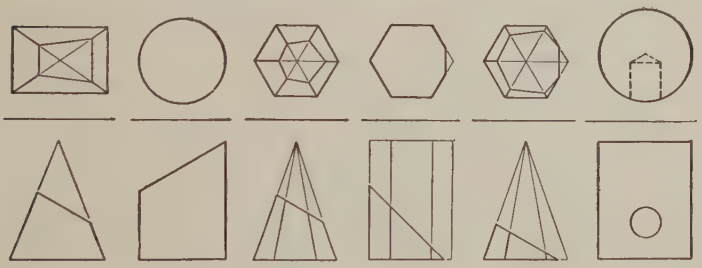


FIG. 101.2. FIG. 101.3. FIG. 102.4. FIG. 102.5. FIG. 102.6. FIG. 102.7.

102.1. (*a*) Show by sketch that the entire surface of a solid may be formed from the developments of the individual surfaces providing these individual surfaces are of developable natures.

(*b*) Must a plane face of a solid be developed when developing the entire surface of a solid? Explain.

102.2. (*a*) When two individual surfaces of a solid intersect along a straight line why is it desirable to join the developments of these surfaces to form a single development?

(*b*) Why is this joining line called a *bending line?*

(*c*) If the surface of a solid is composed of plane faces, is it possible to make a one-piece development of the entire surface?

102.3. (*a*) When forming a one-piece development of several surfaces of

a solid is it permissible to overlap these surfaces on the development? Explain.

(*b*) Demonstrate the use of a *stretch-out line* for drawing a one-piece development of the lateral surface of a rectangular pyramid.

102.4. Copy Fig. 102.4 and develop the entire surface of the solid. Notate fully.

102.5. Copy Fig. 102.5 and develop the entire surface of the solid. Notate fully.

102.6. Copy Fig. 102.6 and develop the entire surface of the solid. Notate fully.

102.7. (*a*) Copy Fig. 102.7 and develop the cylindrical surface.

(*b*) Explain the procedure of developing the hole.

102.8. (*a*) Copy Fig. 102.8 and develop the conical surface.

(*b*) Explain the procedure of developing the hole.

102.9. (*a*) Copy Fig. 102.8 and develop the lateral surface of the pyramid.

(*b*) Explain the procedure of developing the hole.

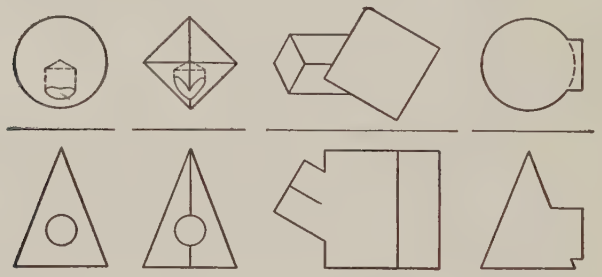


FIG. 102.8. FIG. 102.9. FIG. 102.10. FIG. 102.11.

102.10. (*a*) Copy Fig. 102.10 and describe the intersection of the prisms.

(*b*) Develop the surfaces of the vertical prism.

102.11. (*a*) Copy Fig. 102.11 and describe the intersection of the cylinder and cone.

(*b*) Develop the surface of the cone.

(*c*) Explain how to select points which are used for determining the representation of the intersection and for developing the hole when the *development* is of first importance. Does this selection agree with a selection which best serves the purpose of representing the intersection? Explain.

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